LDPC Coded Soft Forwarding with Network Coding for Two-Way Relay Channel

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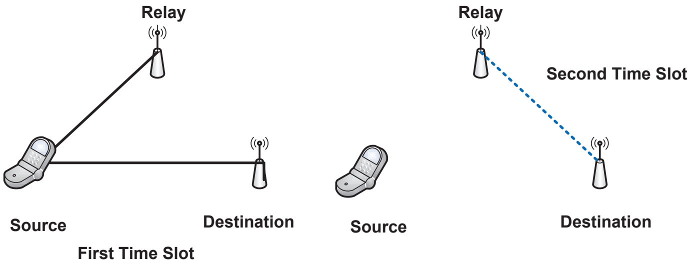
Overview

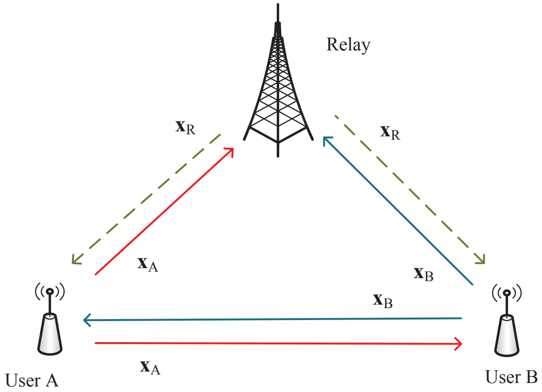
Why soft information relaying?

- The Amplify and Forward (AF) and Decode and Forward (DF) protocols suffer from noise amplification and error propagation, respectively
- In order to combine the advantages of both AF and DF in relay networks, many strategies have been proposed in which soft (reliability) information is transmitted to the destination; this idea is known as soft information relaying (SIR)
- SIR has been shown to be an effective solution which mitigates the propagation of relay decoding errors to the destination

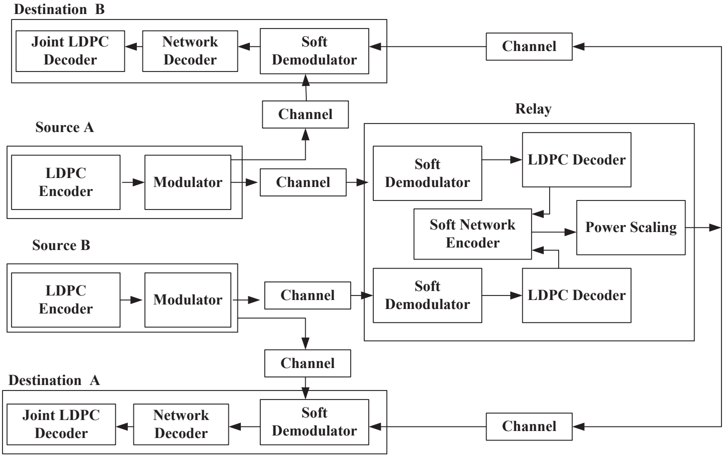


- Soft information indicates the reliabilities or probabilities of the underlying source symbols
- Soft information is often expressed in the form of log-likelihood ratios (LLRs) or soft bits
- As the destination decoder works in the probabilistic domain, the soft information relaying (SIR) protocol complies with the decoder's requirements
- It also gives an idea regarding the reliability of the relay received signal to the destination

- We investigate a *soft network coded two way relay channel* (TWRC) scheme over Rayleigh fading channels with LDPC coding.
- We introduce a model for the effective noise experienced by the soft network coded symbols called the soft scalar model. This model is then used to compute the log-likelihood ratios (LLRs) at the destination.
- For this purpose, an analytical expression is derived for the soft error variance.
- We also introduce a simplified model to calculate the soft error variance which is very easy to compute and adapt on-the-fly.
- Finally, we provide an upper bound of the soft error variance.


Relay
$\mathbf{x}_\mathrm{R}$
$\mathbf{x}_\mathrm{R}$
$\mathbf{x}_\mathrm{A}$
$\mathbf{x}_\mathrm{B}$
$\mathbf{x}_\mathrm{B}$
$\mathbf{x}_\mathrm{A}$
User A
User B

Detailed System Model



- The received signals at each of the nodes in the first and second time slots are

$$\boldsymbol{y}_{iR} = \sqrt{P_i} h_{iR} \mathbf{x}_i + \boldsymbol{n}_{iR},$$

and

$$\boldsymbol{y}_{i\bar{i}} = \sqrt{P_i} h_{i\bar{i}} \mathbf{x}_i + \boldsymbol{n}_{i\bar{i}},$$

where $\boldsymbol{n}_{iR}$ and $\boldsymbol{n}_{i\bar{i}}$ are vectors having i.i.d. real Gaussian (noise) entries with zero mean and variance σ^2_{iR} and $\sigma^2_{i\bar{i}}$, respectively. $i, \bar{i} \in \{A, B\}$ with $i \neq \bar{i}$.

- In the third time slot, the relay employs an LDPC decoder (regular) for decoding (using the parity-check matrix $\boldsymbol{H}$) the noisy codewords received via the user-relay links.

LLR computation

$\lambda_{iR}(x_i^j|\mathbf{y}_{iR}) = \log\left[\frac{P(x_i^j=+1|\mathbf{y}_{iR})}{P(x_i^j=-1|\mathbf{y}_{iR})}\right]$,

where $i \in \{A, B\}$.This computation can be easily performed using an LDPC decoder

Soft network coding operation

The network coding operation can be approximately implemented in the soft domain using the computed *a posteriori* LLR values as

$$\tilde{x}_R^j \approx \mathrm{sign}(\tilde{x}_A^j \tilde{x}_B^j)\min\left(|\tilde{x}_A^j|, |\tilde{x}_B^j|\right),$$

where

$$\tilde{x}_A^j = \lambda_{AR}(x_A^j|\mathbf{y}_{AR})$$

and

$$\tilde{x}_B^j = \lambda_{BR}(x_B^j|\mathbf{y}_{BR}).$$

This can be viewed as the hard decision of the network coded BPSK symbol multiplied by a reliability measurement based on the *a posteriori* LLRs.

- The signal transmitted from the relay can be written as

$$\check{\mathbf{x}}_R = \beta \tilde{\mathbf{x}}_R.$$

- The factor β is chosen to satisfy the transmit power constraint at the relay, i.e., $\mathbb{E}[(\check{x}_R^j)^2] = 1$.
- Thus, the received signal at source i in the third time slot can be written as

$$\mathbf{y}_{Ri} = \sqrt{P_R} h_{Ri} \beta \tilde{\mathbf{x}}_R + \mathbf{n}_{Ri}.$$

We modify the model first introduced in [Jayakody et al.] for the relationship between the correct symbols $x_R^j = x_A^j x_B^j$ and the soft symbols (LLRs) $\tilde{x}_R^j$, but here the model is applied to the LLRs and not to the "soft modulated" symbols:

$$\tilde{x}_R^j = \eta x_R^j + \tilde{n}^j,$$

- where $\tilde{n}^j$ is called the *soft error* variable,
- The constant η is called the *soft scalar* (its effect similar to that of a fading coefficient),
- we choose the value of η which minimizes the mean-square value of the soft error, i.e.,

$$\eta = \mathbb{E}[x_R \tilde{x}_R].$$

- Assuming the soft scalar model, the received signal at each source i in the third time slot can be written as

$$\boldsymbol{y}_{Ri} = \sqrt{P}_R h_{Ri} \beta \eta \mathbf{x}_R + \hat{\boldsymbol{n}}_{Ri},$$

where

$$\hat{\boldsymbol{n}}_{Ri} = \boldsymbol{n}_{Ri} + \sqrt{P_R} h_{Ri} \beta \tilde{\boldsymbol{n}}.$$

- The LLR corresponding to the third time slot transmission is given by

$$\lambda_{Ri}(x_R^j | \boldsymbol{y}_{Ri}) = \log \left[\frac{P(x_R^j = +1 | \boldsymbol{y}_{Ri})}{P(x_R^j = -1 | \boldsymbol{y}_{Ri})} \right] = \frac{2\sqrt{P}_R h_{Ri} \beta \eta}{\hat{\sigma}_{Ri}^2} y_{Ri}^j.$$

where $\hat{\sigma}_{Ri}^2 = \sigma_{Ri}^2 + P_R h_{Ri}^2 \beta^2 \sigma_{\tilde{n}}^2$.

- Note that the *a priori* LLR at source i corresponding to the source $\bar{i}$ is easily calculated as

$$\lambda_{\bar{i}i}(x_{\bar{i}}^{j}|\boldsymbol{y}_{\bar{i}i}) = \frac{2\sqrt{P_{\bar{i}}}h_{\bar{i}i}}{\sigma_{\bar{i}i}^{2}} y_{\bar{i}i}^{j}.$$

- Next, the network decoded soft symbols at source i are computed via

$$\bar{\lambda}_{Ri}(x_{\bar{i}}^{j}|\boldsymbol{y}_{Ri}) = \lambda_{Ri}(x_{R}^{j}|\boldsymbol{y}_{Ri}) \cdot x_{i}^{j}.$$

- At each source i, the parity bit (p) LLRs derived from the relay transmission will be combined with the parity bit LLRs derived from the transmission from source $\bar{i}$

$$\lambda_{i}^{(p)}(\boldsymbol{x}_{\bar{i}}) = \lambda_{\bar{i}i}^{(p)}(\boldsymbol{x}_{\bar{i}}|\boldsymbol{y}_{\bar{i}i}) + \bar{\lambda}_{Ri}^{(p)}(\boldsymbol{x}_{\bar{i}}|\boldsymbol{y}_{Ri}).$$

- First, we note that since the symbols x_R are equidistributed in $\{-1, +1\}$, we have $\mathbb{E}(x_R) = 0$.
- By invoking symmetry of the channel, BPSK modulation, and LDPC decoding process we also have $\mathbb{E}(\tilde{x}_R) = 0$; it follows that $\mathbb{E}(\tilde{n}) = 0$.
- We assume [ten Brink]

$$p_{\tilde{x}_R}(\Lambda | x_R = 1) = p_{\tilde{x}_R}(-\Lambda | x_R = -1),$$

where Λ indicates the network coded LLR at the relay.

Lemma 1

The PDF of the soft error variable conditioned on the network-coded relay symbol satisfies

$$p_{\tilde{n}}(\Lambda|x_R = 1) = p_{\tilde{n}}(-\Lambda|x_R = -1)$$

for all $\Lambda \in \mathbb{R}$.

Corollary 1

The PDF of the soft error variable possesses even symmetry, i.e.

$$p_{\tilde{n}}(\Lambda) = p_{\tilde{n}}(-\Lambda)$$

for all $\Lambda \in \mathbb{R}$.

Lemma 2

This lemma proves that the soft scalar η is independent of conditioning on x_R,i.e.

$$\mathbb{E}(\tilde{x}_R|x_R = +1) = -\mathbb{E}(\tilde{x}_R|x_R = -1) = \eta.$$

Analytical form of soft error variance

The soft error variance may be expressed as

$$\begin{aligned}
\mathbb{E}(\tilde{n}^2) &= \mathbb{E}((\tilde{x}_R - \eta x_R)^2), \\
&= \mathbb{E}(\tilde{x}_R^2) - 2\eta\mathbb{E}(x_R\tilde{x}_R) + \eta^2, \\
&= \mathbb{E}(\tilde{x}_R^2) - \eta^2 \,.
\end{aligned}$$

Note that this expression can be used to directly estimate the soft error variance, as the two terms involved can be estimated at the receiving node.

Upper bound on the soft error

$$\sigma_{\tilde{n}}^2 \leq \mathbb{E}(\tilde{x}_R^2),$$

where η^2 is a positive value. Note that the upper bound can be computed without the knowledge of actual or estimated information bits.

Assumption 1

We assume as in [ten Brink] that the variance of the LLR, conditioned on the transmission of the symbol $+1$, is equal to twice its mean, i.e.,

$$\mathbb{E}((\tilde{x}_R - \mu^+)^2 | x_R = +1) = 2\mu^+ \,.$$

where $\mu^+ = \mathbb{E}(\tilde{x}_R | x_R = +1)$.

Theorem 1

Under the Assumption 1, the soft error variance can be expressed as

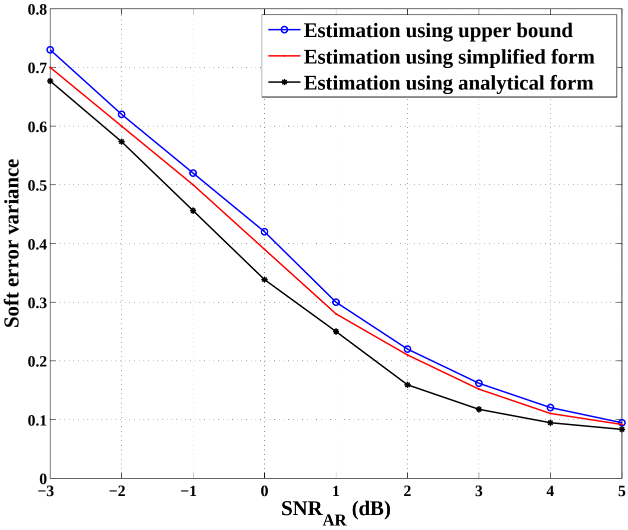
$$\sigma_{\tilde{n}}^2 = 2\eta.$$

- When Assumption 1 holds, Theorem 1 provides a very computationally efficient means to estimate the soft error variance.
- This expression is easy to compute on-the-fly, and has very low implementation complexity.

We compare soft decode and forward (SDF) scheme with hard decode and forward (DCF) scheme in a flat Rayleigh fading.

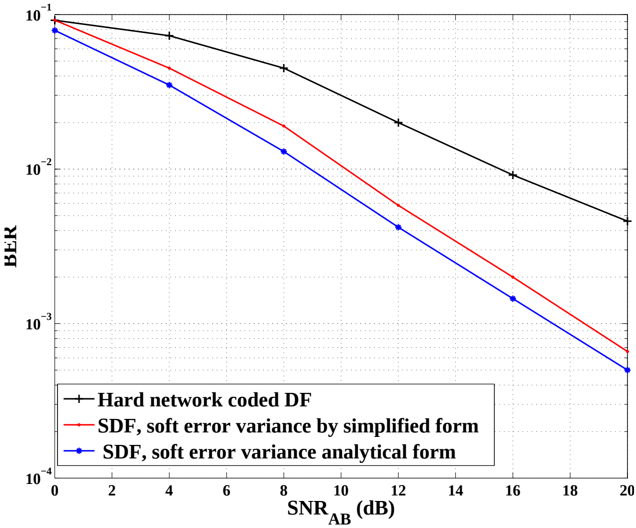
We have

- the relevant dimensions of the parity-check matrix are $N = 816$ and $K = 408$,
- the simulations assume BPSK,
- power normalization of $P_A = P_B = P_R = 1$,
- all simulations assume $\mathrm{SNR}_{AR} = \mathrm{SNR}_{BR}$,
- set the source-relay link *SNR*s to be 3dB.

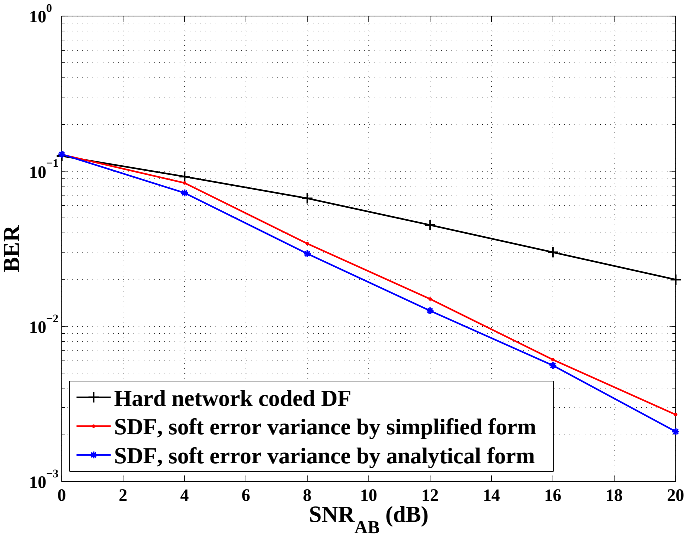


Note that the upper bound can be computed and tracked without knowledge of the data symbols at the relay, but other two results are a more accurate fit to the soft scalar model.

BER performance of the proposed LDPC coded SDF scheme



A comparison of the BER performance of the proposed SDF scheme with that of hard network-coded DF relaying. The source-relay link SNRs were both kept constant at 3dB.



A comparison of the BER performance of the proposed SDF scheme with that of hard network-coded DF relaying. The source-relay link SNRs were both kept constant at 3dB.

- This paper investigates an LDPC code based SDF relay protocol in the TWRC.
- We present a new analytical expression for the soft error variance, a simplified expression, and an upper bound for the soft error variance.
- The derived analytical expression facilitates the estimation of the soft error variance without having to access the actual or estimated information signal of the sources.
- This makes the LLR computation at the destination more precise as well as adaptable to changing channel conditions.

References

Motivation

System Model

Soft Forwarding

Calculation of LLR

Soft Error Variance Analysis

Simulation Results

Summary

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Thank You

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