

# Fractionally Spaced Self-Interference Canceler for Full-Duplex Communication Systems

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**Abstract**—Requiring a single frequency band for transmission, full-duplex (FD) systems can ideally double the spectral efficiency. However, in addition to the information signal, radiated from user's own transmit antenna generates a strong self-interference (SI). To eliminate effects of SI, active and passive cancellation techniques are used in the literature. However, omitted in these studies is the fact that the time dispersion in the SI channel may cause an error floor. In this paper, it is verified that the SI channel is time dispersive via measurements. Later, we present an analysis in order to determine the error performance. Following these results, a fractionally spaced SI canceler is proposed to eliminate the error floor. We verify our analytical results via simulations, showing that the proposed SI canceler can critically enhance the performance of FD systems.

**Index Terms**—Full-duplex systems, self-interference canceler, time dispersive channel, oversampling.

## I. INTRODUCTION

Wireless communication systems use half-duplex transmission techniques, which can enable only the transmission or the reception of data at a specified time interval or frequency band. However, taking into account the ever increasing demand of data traffic, efficient usage of radio sources is crucial. Full-duplex (FD) systems are developed as a method to increase the spectral efficiency at the same frequency channels at the same time interval. However FD transmission has not yet been used in communication systems practically due to the generated self-interference (SI) by the local transmit antenna since the SI can be more powerful (15-100 dB) than the transmission signal at the receiving antenna [1].

In order to design an operational FD system, using only passive cancellation techniques would not be sufficient to eliminate the effects of the SI. Passive and active cancellation techniques are jointly applied in FD systems to remove destructive effects of the SI on the system's error performance [1–5]. In addition to active and passive cancellation techniques, antenna separation is also proposed in [1,2], where more than two antennas are required without changing the diversity order. However, as stated in [1,2], such a system is not suitable for wide-band communications since the phase offsets of transmission signals radiated from transmit antennas may drift. On the other hand, a practical example of FD systems is in cognitive radios, which is a solution for low spectrum utilization due to fixed frequency partition. The main problem of cognitive radio networks (CRN) is that the spectrum can not be detected by secondary user during transmitting, causing interference and

delay for primary users. In [4], to overcome this case, an FD scheme is proposed for CRN by using antenna, analog and digital cancellation techniques. Asynchronous FD system is performed by digital and RF cancellation [5].

In the literature it is shown that solely using active and passive cancellation techniques may not be sufficient. In [6], a baseband transceiver is designed by using an echo canceler for an FD system using the IEEE 802.11a/g standard, and it is shown that signal-to-interference-and-noise ratio (SINR) can be increased. In [7], an FD communication interface is embedded in tele-presence systems by using aspect of echo cancellation. However, the effects of the SI and corresponding error performance is not modeled and analyzed in [6,7].

Noting that the severe performance degrading effects of the SI for FD systems, in this paper we aim to quantify and eliminate the effects of the SI channel. We model the SI channel as time dispersive, and analyze the performance loss due to residual interference for the first time in the literature. The time dispersion of the SI channel is expected from the works on electromagnetic (EM) side channel attacks when authors measured the EM emanations by placing coils near the chip in the CMOS devices and showed that the measured signals consist of aggregation of the radiated signals [8]. To verify this conclusion, we used a Universal Software Radio Peripheral (USRP) testbed to estimate a narrow-band SI channel with pilot symbols at 2.4 GHz. From test results, we observe that the channel is time dispersive with an exponential distribution, even for narrow-band signals. After verifying the fact that the SI channel can be time dispersive, we provide an analysis to quantify the degrading effects of the time dispersion and show that the residual interference may cause an error floor. Accordingly, inspired by the echo cancellation techniques for wired communications and fractionally spaced equalization as in [9–11], we propose a fractionally spaced SI canceler for the FD receiver design, and provide the associated error performance. Our analytical results are also verified by simulation results. We demonstrate that depending on the dispersion of the channel, there is a trade-off between the oversampling rate and the receiver performance, and an error floor may become inevitable if the channel is not modeled with the required time accuracy. This paper is organized as follows. In Section II, the system model and performance analysis are presented. The fractionally spaced SI canceler is proposed in Section III, along with performance analysis for

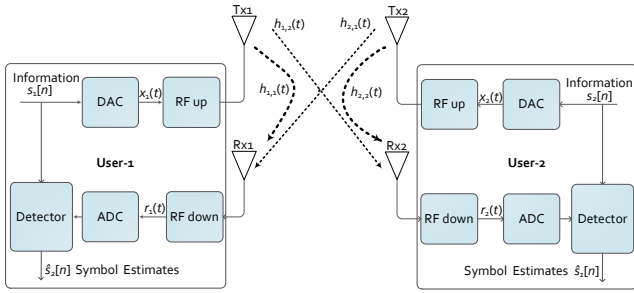


Fig. 1. Block diagram of the full-duplex communication system.

ideal and imperfect channel estimates. In Section IV, channel measurements are given and error performance is investigated with simulations and theoretical calculations. Finally, the conclusions are discussed in Section V.

## II. SYSTEM MODEL AND PERFORMANCE ANALYSIS

The block diagram of an FD system is shown in Fig. 1 which is similar to the model in [5], where users have distinct antennas for transmission and reception that are closely spaced.

### A. System Model

In the system shown in Fig.1, information symbols,  $s_1[n]$  and  $s_2[n]$  that are uncorrelated and equally likely selected from a finite  $M$ -ary set, are transmitted by user-1 and user-2, respectively with using the same frequency band. Let the output of the digital to analog converter (DAC) and RF down-conversion is denoted by  $x_i(t)$  and  $r_i(t)$  for  $i = 1, 2$  for user-1 and user-2, respectively. Assuming that users are sufficiently spaced apart, transmission channels between them, namely  $h_{1,2}(t)$  and  $h_{2,1}(t)$  are modeled as Rayleigh fading, where  $h_{1,2}(t) = v_{1,2}\delta(t)$  and  $h_{2,1}(t) = v_{2,1}\delta(t)$ , the channel taps are *i.i.d.* and  $v_{i,n} \sim \mathcal{CN}(0, 1)$  for  $n^{\text{th}}$  realization, (i.e.  $n^{\text{th}}$  symbol).

The SI channels,  $h_{1,1}(t)$  and  $h_{2,2}(t)$ , are located within a close proximity of corresponding receivers, hence their average energies are expected to be significantly higher than those of the transmission channels. Motivated by this, we investigate performance results by using the residual interference which can cause an error floor in the system due to time dispersion of these higher energy channels. In the most generalized form, the baseband SI channel can be modeled in the [12]

$$h_{i,i}(t) = \sum_{p=-\infty}^{+\infty} \alpha_{i,p} \delta(t - \tau_{i,p}) \quad (1)$$

where  $\alpha_{i,p}$  and  $\tau_{i,p}$  are the attenuation and the delay of the  $p^{\text{th}}$  SI channel path for user  $i$ , respectively. Since circuit components of a receiver are securely attached, time variations in the received SI signal is not considered. The time dispersion of the SI channel can be included in the system's performance analysis by using (1). In conventional FD receivers, the RF downconverted received analog signal can be modeled as [3]

$$r_i(t) = h_{1,i}(t) * x_1(t) + h_{2,i}(t) * x_2(t) + w_i(t) \quad (2)$$

where  $w_i(t)$  represents the complex additive white Gaussian noise with double sided power spectral density of  $N_0/2$ , and  $*$  represents the convolution operator.  $r_i(t)$  is then digitized using the analog to digital converter (ADC) block every  $T_s$  seconds, where  $T_s$  is the symbol period.

Our goal is to determine the error rates, associated with the information signals estimates  $\hat{s}_i[n]$  and determine the error floor that can be encountered because of the residual SI. Note that EM side-effect attacks, investigated in [8] also makes use of high energy levels and time dispersiveness of the SI channels. However, recent studies, simply modeling the SI channel as a Ricean fading channel model in [2], do not consider the effect of residual SI in the receivers due to the of time dispersive nature of these channels. This residual interference needs to be taken into account, since the error performance of the FD communication system may dramatically deviate from the expected levels.

### B. Performance Analysis

As for the performance analysis, we start by defining a minimum required sampling factor,  $K$ , as the smallest integer value that satisfies  $d_{i,p} = \tau_{i,p}K/T_s$  and  $d_{i,p} \in \mathbb{Z}^+$ ,  $\forall i, p$ . If the sampling rate is sufficiently high, channel taps can be modeled as discrete variables. With this goal, we define  $n$  and  $k$  that represent sampling periods of  $T_s$  and  $T_s/K$ , respectively. The  $i^{\text{th}}$  user's discretized time dispersive SI channel of length of delay spread  $P$  can now be expressed as

$$h_{i,i}[k] = \sum_{p=0}^{P-1} \alpha_{i,p} \delta[k - d_{i,p}]. \quad (3)$$

Digital cancellation systems rely on the reconstruction of SI and elimination of its effects from the received signal. Let the energy of the SI be defined as  $I_{est,i} = E[\sum_n |s_i[n] * \tilde{h}_{i,i}[n]|^2]$  that is sampled with frequency  $1/T_s$ . Here, the estimated discrete channel is denoted by  $\tilde{h}_{i,i}[n]$ . However, assuming that ideal channel taps can be obtained, there is still a difference between estimated and the real channel due to sampling rates when  $K > 1$ . The estimated SI channel can be modeled as

$$\tilde{h}_{i,i}[k] = \sum_{p=0}^{P-1} b_{p,i} \alpha_{p,i} \delta[k - p] \quad (4)$$

where

$$b_{p,i} = \begin{cases} 1, & \text{mod}(p, K) = 0 \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

In order to model the effects of interference we consider the model given in Fig. 2, where the input symbol  $s_i[n]$  is upsampled by a factor of  $K$ . Upsampled signal is then transmitted through a unit energy rectangular window  $w[k]$ , to obtain

$$o_i[k] = \frac{1}{\sqrt{K}} s_i[\lfloor k/K \rfloor] \quad (6)$$

where  $n$  is equal to  $\lfloor k/K \rfloor$  and  $\lfloor \cdot \rfloor$  is the floor function which map a real number to the largest previous integer. The actual interference, including the effects of the time dispersive

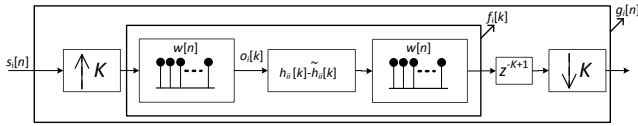


Fig. 2. Equivalent oversampled system model to analyze residual interference.

channel is  $I_{act,i} = E[\sum_k |o_i[k] * h_{i,i}[k]|^2]$ . Here, the total length of the oversampled symbol vector is  $K$  times that of the information symbols.

The channel will cause residual interference that can be obtained by  $h_{i,i}[k] - \tilde{h}_{i,i}[k]$ . Hence  $o_i[k]$  is passed through this filter. In order to sample at every  $K$  symbols for the detector, another unit energy rectangular window  $w[k]$  is used along with a delay of  $K - 1$  samples, so all  $K$  symbols are summed together to obtain the  $n^{\text{th}}$  sample. Following the polyphase decomposition, in order to generate a stationary output signals [13], the residual interference can be shown to be  $s_i[n] * g_i[n]$ , where

$$g_i[n] = f_i[(n+1)K - 1] \quad (7)$$

and  $f_i[k] = w[k] * (h_{i,i}[k] - \tilde{h}_{i,i}[k]) * w[k]$ . Hence, it can be shown that the average energy of residual interference defining the average symbol energy as  $S_i = E[|s_i[n]|^2]$  is

$$I_{res,i} = S_i \sum_{n=0}^{\lfloor P/K \rfloor} |g_i[n]|^2. \quad (8)$$

Representing the noise power as  $N = N_0B$ , where  $B$  is the transmission bandwidth ( $B \propto 1/T_s$ ), it can be shown that the average SINR of this system at the  $i^{\text{th}}$  user,  $\tilde{\gamma}_i$ , is

$$\tilde{\gamma}_i = \frac{S_i}{I_{res,i} + N}. \quad (9)$$

The SINR expression in (9) can be used to evaluate bit error rate (BER) performance of M-QAM constellations by using moment generating function of  $\tilde{\gamma}_i$ , according to [12]

$$P_e(M, \tilde{\gamma}_i) = \frac{4}{\pi} \left(1 - \frac{1}{\sqrt{M}}\right) \int_0^{\pi/2} \left(1 + \frac{1.5\tilde{\gamma}_i}{\sin^2 \phi}\right)^{-1} d\phi - \frac{4}{\pi} \left(1 - \frac{1}{\sqrt{M}}\right)^2 \int_0^{\pi/4} \left(1 + \frac{1.5\tilde{\gamma}_i}{\sin^2 \phi}\right)^{-1} d\phi. \quad (10)$$

This integral expressions can be numerically calculated. Note that this analysis can easily be generalized to other constellations. When the channel is time dispersive ( $K > 1$ ), the SI cannot be canceled completely, causing an error floor  $P_e(M, S_i/I_{res,i})$  at high signal-to-noise ratio (SNR) values. In the following section, we propose a fractionally spaced interference canceler in order to eliminate the residual interference to avoid an error floor.

### III. FRACTIONALLY SPACED SELF INTERFERENCE CANCELER

Having shown that time dispersion may critically affect the error performance of FD communication systems, we propose

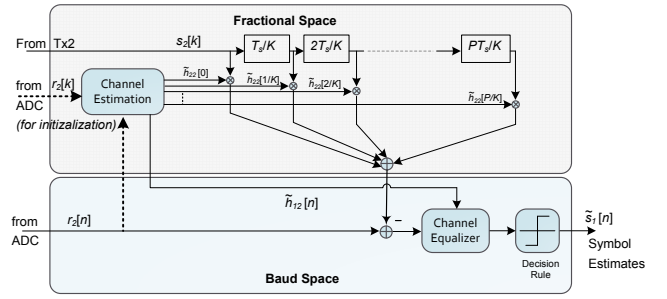


Fig. 3. The proposed fractionally spaced interference canceler and symbol detector.

a fractionally spaced SI canceler, inspired by classical echo cancellation techniques in the literature [9, 10].

The block diagram of the proposed detector is shown in Fig. 3. There are two different sampling rates are used as baud and fractional rate with sampling ratio of  $\tilde{K}$  (i.e. symbols are sampled every  $T_s/\tilde{K}$  seconds). The channel estimation block operates at the fractionally sampled space, in order to determine the SI channel accurately in (3). A time division multiplexing based channel estimation protocol can be implemented between users so that in the first time slot user-1 transmits pilot symbols for estimating the SI and transmission channels by user-1 and user-2. The procedure can be repeated for the user-2 in the following time slot.

Once the SI channel is estimated, a tapped delay line, composed of unit delays,  $T_s/\tilde{K}$ , is implemented in order to replicate SI. Following this, SI signal is transferred to the baud space for cancellation. After cancellation process, the channel equalizer is implemented for the transmission channel. Then the received signals are fed into the decision rule block in order to obtain received symbols correctly.

#### A. Performance Analysis

We also used the model in Fig. 2 with different filters and consider  $\tilde{K}$  as the upsampling and downsampling ratio. The estimated SI channel with oversampling factor of  $\tilde{K}$  can be modeled as

$$\hat{h}_{i,i}[k] = \sum_{p=0}^{P-1} c_{p,i} \alpha_{p,i} \delta[k-p] \quad (11)$$

and in this case the channel taps are

$$c_{p,i} = \begin{cases} 1, & \text{mod}(p, \tilde{K}) = 0 \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

since channel estimate can be obtained with the resolution  $\tilde{K}$ . In this formulation, we consider the fact that  $\tilde{K}$  can be smaller than the required sampling factor of  $K$ . Hence the residual interference can be again calculated according to (7), where  $f_i[k] = w[k] * (h_{i,i}[k] - \hat{h}_{i,i}[k]) * w[k]$ . Note that when  $\tilde{K} > 1$ , it can be observed that the energy of  $f_i[k]$ , consequently the energy of the residual interference can be reduced. When  $\tilde{K} = K$ , then  $c_k = 1, \forall k, g_i[n] = 0, \forall n$ ,

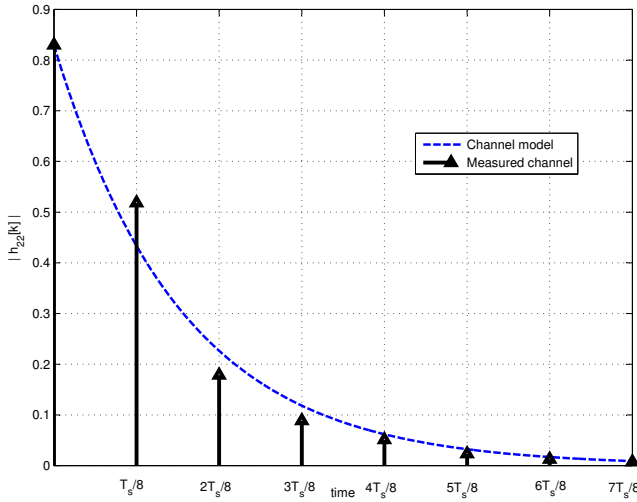


Fig. 4. Channel estimation measurement results ( $T_s = 5 \mu\text{sec.}$ ,  $K = 8$ ,  $P = 8$ ,  $\alpha = 0.8296$  and  $\beta = 0.6495$ ).

hence  $I_{res,i} = 0$ , i.e. residual interference can completely be canceled.

Note that when  $\tilde{K} < K$ , although the overall residual interference is reduced, we may still observe an error floor  $P_e(M, S_i/I_{res,i})$ , due to insufficient sampling rate at the receiver. Hence selection of the correct sampling ratio is critical, as will be shown in Section IV. In order to quantify the effects of the oversampling at the receiver we define the SI suppression ratio as

$$SI_{SR} = \frac{I_{est,i}}{I_{res,i}} \quad (13)$$

that acts as an indicator of the ratio of the energy of the canceled SI components to the residual SI, that is observed due to the unestimated channel components at the receiver.

#### B. Effects of Imperfect Channel Estimates

In the expressions of previous sections it is perfect channel state information (CSI) was available both transmission and SI channels. However, this assumption may not be realistic since the error performance may significantly deviate from the target rate depending on the channel estimation error. Modeling the estimation error of channel coefficients is used in [14] the channel taps can be represented as

$$\begin{aligned} \tilde{h}_{1,i}[k] &= h_{1,i}[k] + h_{e,1,i}[k] \\ \tilde{h}_{2,i}[k] &= h_{2,i}[k] + h_{e,2,i}[k] \end{aligned} \quad (14)$$

where the estimation errors  $h_{e,1,i}[k]$  and  $h_{e,2,i}[k]$  are i.i.d. Gaussian distributed having variances of  $\sigma_{e,1,i}^2 = \rho_{e,1,i}E[|h_{1,i}|^2]$  and  $\sigma_{e,2,i}^2 = \rho_{e,2,i}E[|h_{2,i}|^2]$  respectively and  $\rho_{e,1,i}$  and  $\rho_{e,2,i}$  are the corresponding percentile coefficients, for the  $i^{\text{th}}$  user. Hence in the presence of channel estimation errors average SINR becomes

$$\bar{\gamma}_{e,i} = \frac{S_i}{I_{res,i} + \sigma_n^2 + \sigma_{e,1,i}^2 + \sigma_{e,2,i}^2}. \quad (15)$$

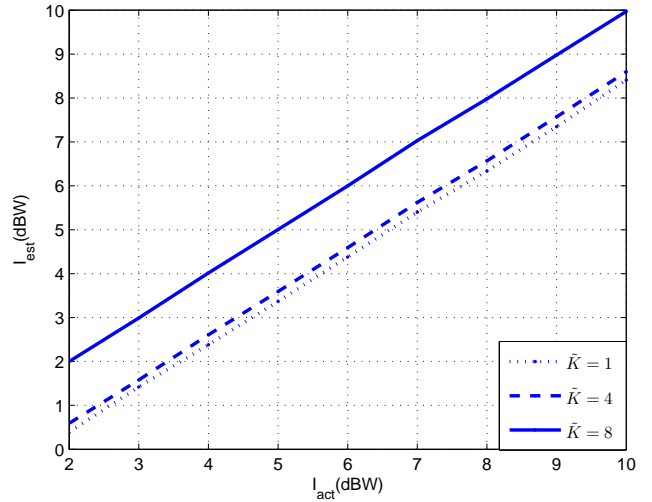


Fig. 5. Equivalent oversampled system model to analyze residual interference.

Here,  $\rho_{e,2,i}$  parameter effects more than  $\rho_{e,1,i}$  on the error performance since it is expected to have higher energy than the transmission channel. Hence, in presence of channel estimation errors,  $P_e(M, \bar{\gamma}_{e,i})$  given in (12) can be used to obtain theoretical error rates for M-QAM constellations.

## IV. TEST AND SIMULATION RESULTS

### A. Channel Estimation Tests

In order to design the receiver in accordance with the properties of SI channel, we use USRP-N210 kits for estimating SI channel. The distance between transmit and receive antennas is set to 10 cm and the radio is operated as FD mode. Narrow-band signals ( $B = 200$  kHz) are used at the 2.4 GHz with  $\tilde{K} = 8$ . Test results show that the SI channel has an exponential decay as shown in Fig. 4. Here, the SI channel can also be modeled as  $|h_{i,i}[k]| = \beta e^{-\lambda k}$ . In Fig. 4, the parameters for unit energy channel are  $\beta = 0.82960$ ,  $\lambda = 0.6495$ .

We also calculated the canceled interference levels according to the energy of the SI channel as shown in Fig. 5, versus different sampling ratios. The classic FD receiver  $\tilde{K} = 1$  cancels 8.4 dB of the interference when  $I_{act,i} = 10$  dB and  $SI_{SR}$  is equal to 3.4 dB. It can be observed from this figure that the performance of the canceler improves as  $\tilde{K}$  increases and  $\tilde{K} = 1$  which has the worst performance. When  $\tilde{K} = 4$ , 8.6 dB of the interference can be canceled again for 10 dB and  $SI_{SR}$  is 4.2 dB for this case. For  $\tilde{K} = 8$ , 9.9 dB interference can be canceled. The cancellation slope is equal to 1 and for  $\tilde{K} = 8$ ,  $SI_{SR} \rightarrow \infty$ . Note that the residual interference may be even higher for lower  $\lambda$ . In accordance with the analysis, it can be observed that  $SI_{SR}$  increases as  $\tilde{K}$ .

### B. Error Performance Analysis

In the simulations, 16-QAM constellation is used with unit energy. The SI channel parameters are set to  $\beta = 0.46$ ,  $\lambda = 2$  to easily visualize the error performance. The error floor can be observed from Fig. 6 as the interference power becomes more

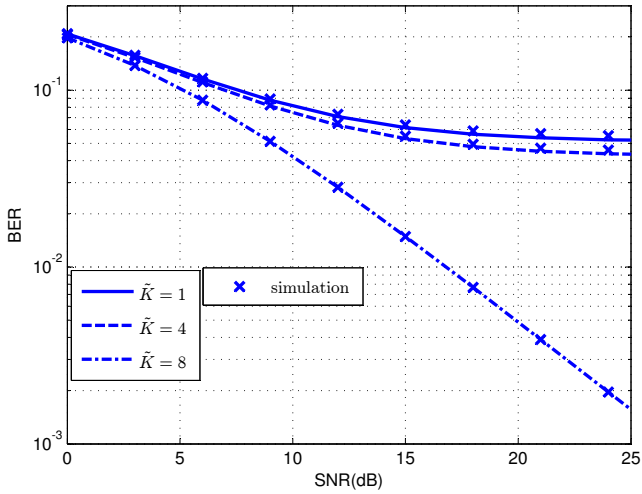


Fig. 6. The theoretical and simulation bit-error rate performances of 16-QAM versus different oversampling ratios at the receiver.

dominant than the noise power. The length of time spread channel is  $P = 8$ . In line with measurement results, we set  $K = 8$ .

From Fig. 6, we see that when sampling at baud rate where  $\tilde{K} = 1$ , the error performance is not acceptable, and error floor is at  $5.1 \times 10^{-2}$ . When sampling rate is equal to four times the baud rate, the BER performance improves. For this case error floor is reduced to  $4.3 \times 10^{-2}$ . If the sampling rate is selected as properly (i.e.  $\tilde{K} = K$ ) the SI can be completely canceled and the error floor is eliminated. Hence, to communicate with FD radios, the delay spread of SI should be properly addressed, and to cancel the effect on BER, the sampling rate should be correctly determined. In Fig. 7, the error performance of FD radio with different channel estimation errors has been shown to observe the effect of  $\rho_{e,1,i}$  and  $\rho_{e,2,i}$ . The SNR value is 10 dB and the sampling rate is equal to baud rate. If there are no estimation error ( $\rho_{e,1,i}$  and  $\rho_{e,2,i}$  are equal to zero) the BER performance is the same as given in Fig. 6.

## V. CONCLUSION

In this paper, we demonstrated via channel estimation measurements that the SI channel in FD systems can be time dispersive and proposed a new fractionally spaced SI canceler to improve the error performance that eliminates residual interference, time; dispersiveness of the channel. The results of studies are, if the received signal is sampled at baud sampling rate of lower sampling than the local channel's characteristics, the SI channel components can not be estimated completely, hence unestimated local channel coefficients cause residual interference, the BER performance gets deteriorated and an error floor is encountered. We formulated this residual interference and verified the presented error performance analysis with simulations under different oversampling ratios at the receiver for M-QAM modulations. We also highlighted the importance of accurate channel estimates with simulation and analytical results. To conclude, in the FD radios, according

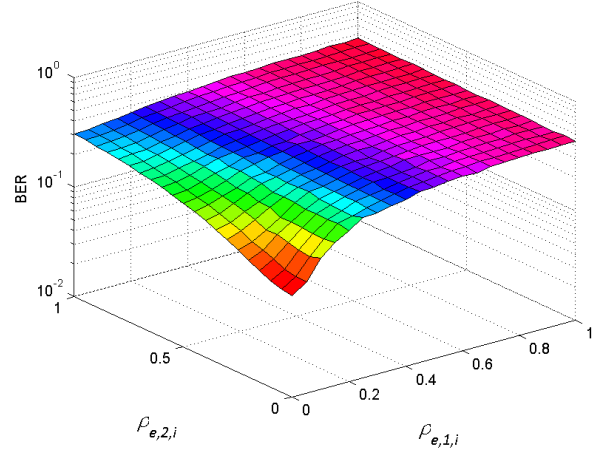


Fig. 7. Bit-error rate performances of 16-QAM versus different channel estimation errors,  $\rho_{e,1,i}$  and  $\rho_{e,2,i}$  (SNR=10 dB).

to the oversampling rate must be chosen the length of local channel to discard the SI completely.

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