RESEARCH REPORT

Performance of Location Estimation Systems

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National regulation authorities require mobile network operators to determine locations of mobile subscribers. These operators also offer several location based services and have to keep track of the accuracy of the location estimates.

In this report, the accuracy of the location estimates is investigated based on the data and decision fusion processes that is possible in location estimation systems. The pdf of the radial estimation error is given by making use of the central limit theorem and it is shown that the analysis provides a good approximation for the distributions.

Simulation codes used in this report are available upon request.

I. INTRODUCTION

Location estimation is a critical component of mobile communication systems. The performance of the location estimation systems need to be accurately estimated. There are two main reasons behind this. As the first reason, governmental regulation agencies impose compliance requirements to mobile network operators for the location estimation performance. For example, Federal Communications Commission (FCC) mandates operators to provide mobile users' locations [1]. The second reason is that the mobile network operators are providing several location based services and the accuracy of these services are their responsibility.

There are several methods in the literature to estimate the location of mobile station. Frequently used ones are received signal strength (RSS), time of arrival (TOA), angle of arrival (AOA) and time difference of arrival (TDOA) [2,3]. Different mobile location estimation techniques with varying levels of accuracy levels were proposed by researchers. Most of these solutions make use of one of the above-mentioned techniques. However using a single measurement might not be sufficient for desired accuracy levels. It has been shown that using a fusion of these techniques can increase the accuracy of the location estimates. For example, a hybrid TDOA/RSS combination system is implemented in [4] and hybrid TOA/TDOA is implemented in [5]. With expanding use of antenna arrays and multi-carrier communication systems that provide robustness against severe fading conditions [6], location estimation systems have plenty of measurement data to make use of. The probability density function (pdf) of radial estimation error for multiple measurement cases is shown in this report. Via simulation results, we test the Gaussianity of estimates. A methodology to obtain the location estimation accuracy is described. Based on this methodology, the compliance requirements that are introduced by the FCC is checked for the given estimation system.

II. LOCATION ESTIMATION SYSTEM AND PBS

Let the actual and the estimated 2-D coordinates of the mobile station be represented by (x, y) and $(\hat{x}, \hat{y})$, respectively. A location estimation system aims to obtain estimates $(\hat{x}, \hat{y})$, that are closest to (x, y) by making use of received signals at some known reference positions. The system model is shown in Fig. 1.

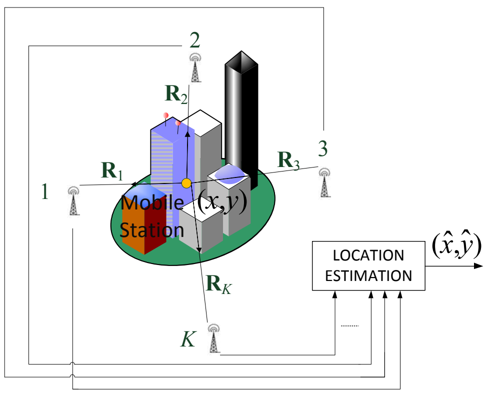

Figure 1. System model.

Let the number of reference points (depicted in Fig. 1 as base stations in an urban environment) be denoted by K. Assume that each reference point is equipped with an M element antenna array. Let the number of subcarriers and time domain measurements to be used by the mobile station to be located be represented by N. Let the $M \times N$ dimensional measurement matrix of k^{th} reference point be denoted by $\mathbf{R}_k$. The location estimation systems in the literature make use of the field measurement set $\{\mathbf{R}_k\}_{k=1}^{K}$ and estimate the position of the mobile station [7, 8].

In order to obtain a position estimate three distance estimates, d_1, d_2 and d_3 with K=3. (Fig. 2 (a)) or two angle estimates ϕ_1 and ϕ_2 with K=2 are sufficient (Fig. 2 (b)). Frequently, the estimation system explicitly determines AOA, TOA, TDOA and geometry. In addition to these techniques RSSI estimates can be measured and compared with a signal strength model that makes use of the distance between reference point and the mobile station [7, 8].

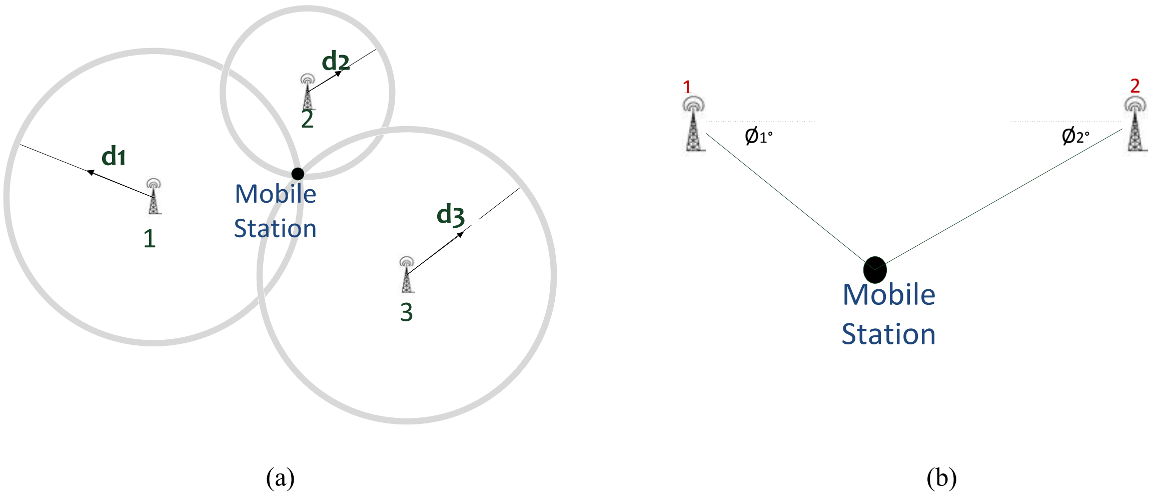

Figure 2. Estimation geometry (a) Distance based estimation, (b) Angle based estimation.

By making use of multiple measurements of multiple carriers and antenna outputs, we can receive spatially diverse signals, which can improve the performance of the location estimation system.

A. On the Gaussianity of Coordinate Estimates

Considering the system model given in Fig. 1, the location estimation block can obtain at $N \times K$ angle estimates of using the AOA of each subcarrier and reference point during the transmission of a single symbol. The block can also extract $M \times K$ TOA or TDOA estimates. The number of RSSI estimates of a symbol is $M \times N \times K$. Hence, the location estimate system can make use of $O(M \times N \times K)$ measurements. Typical configuration in the multi-carrier Long Term Evolution (LTE) standard is expected as $M = 4$, $N = 1024$ and $K = 3$ [9], hence it is possible to obtain in excess of 12000 position estimates.

Let the number of position estimates that can be obtained by the location estimation system be represented by P. After obtaining these estimates in x and y directions, one can obtain the minimum mean square estimate for $(\hat{x}, \hat{y})$ by averaging P estimates as $\hat{x} = \sum_{i=1}^{P} \hat{x}_i / P$ and $\hat{y} = \sum_{i=1}^{P} \hat{y}_i / P$. Note that this estimation is valid when the channel is stationary during at least one symbol duration. This assumption is frequently utilized in the literature for channel estimation purpose [6] and does not impose any additional constraints to the location estimation system.

The central limit theorem states that as P becomes large, the cumulative distribution function (cdf) becomes the cdf of a Gaussian random variable [10]. Hence, we can approximate the cdf's of $\hat{x}$ and $\hat{y}$ with that of Gaussian random variables. The joint pdf of $(\hat{x},\hat{y})$ can then be approximated as

$$f_{\hat{x},\hat{y}}(\hat{x},\hat{y}) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}\exp\left(\frac{(\hat{x}-x)^2}{\sigma_x^2}\right)\times\exp\left(\frac{2\rho(\hat{x}-x)(\hat{y}-y)}{\sigma_x\sigma_y}+\frac{(\hat{y}-y)^2}{\sigma_y^2}\right). \quad (1)$$

where ρ represents the correlation coefficient of $\hat{x}$ and $\hat{y}$, σ_x and σ_y are the variances in x and y dimensions, respectively.

Estimation errors x and y directions can be calculated as $e_x = x-\hat{x}$ and $e_y = y-\hat{y}$. Since (x,y) is a pair of scalars, the estimation errors in both directions can be shown to be zero mean Gaussian random variables. Note that they may or may not be independent depending on the location estimation methodology used in the system. If they are independent then $\rho = 0$.

III. PROBABILITY DENSITY FUNCTION OF THE ESTIMATION ERROR

The performance of location estimation systems is traditionally measured through the radial estimation error. Regulatory organizations determine the accuracy versus probability limits in when introducing new requirements for the mobile system operators [1]. Radial estimation error can be calculated as

$$R = \sqrt{e_x^2+e_y^2}\,. \quad (2)$$

Let the angle between x and y components of the estimation error be represented as $\theta = \tan^{-1}(e_y/e_x)$. Hence $e_x = R\cos\theta$ and $e_y = R\sin\theta$. The joint pdf of R and θ can be obtained using the Jacobian of the transformation [10, p. 228]. For correlated Gaussian random variables, the joint probability density function is

$$f_{R,\Theta}(r,\theta) = \frac{r\times\exp\left\{\frac{-1}{2(1-\rho^2)}\left[\frac{(r\cos\theta)^2}{\sigma_x^2}\right]\right\}}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}\times\exp\left\{\frac{-1}{2(1-\rho^2)}\left[\frac{(2\rho r^2\cos\theta\sin\theta)}{\sigma_x\sigma_y}+\frac{(r\sin\theta)^2}{\sigma_y^2}\right]\right\}. \quad (3)$$

For the case that $\sigma_x = \sigma_y = \sigma$, (3) is reduced to

$$f_{R,\Theta}(r,\theta) = \frac{r\times\exp\left\{\frac{-1}{2(1-\rho^2)}\left[\frac{r^2(1-\rho\sin(2\theta))}{\sigma^2}\right]\right\}}{2\pi\sigma^2\sqrt{1-\rho^2}}. \quad (4)$$

Note that (4) can be observed when reference points are uniformly distributed along x and y coordinates along the coverage area.

Integrating the (4) over θ, the pdf of R can be obtained as

$$f_R(r) = \frac{r}{\sigma^2\sqrt{1-\rho^2}}\exp\left(-\frac{r^2}{2\sigma^2\left(1-\rho^2\right)}\right)I_0\left(\frac{r^2\rho}{2\sigma^2\left(1-\rho^2\right)}\right), \qquad (5)$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind. In the case of independent estimates (i.e. $\rho = 0$) R is a Rayleigh random variable and the corresponding pdf is reduced to

$$f_R(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right). \qquad (6)$$

Note that a closed form expression for $f_{R,\Theta}(r,\theta)$ does not exists for the case $\sigma_x \neq \sigma_y$, however the pdf of the radial error can easily be obtained using numeric integration of (3) over θ.

IV. Analysis and Simulation Results

In this section, simulation results are presented to demonstrate the difference between location estimates of system that make use of single measurement and multiple measurements. The location estimation system makes use of TOA and RSSI measurements for both single carrier [8] and multi-carrier systems [9]. The mobile station is assumed is located at (12,150) in the urban propagation environment shown at Fig. 1 of [8].

The measurement error in TOA measurements in NLOS distance is assumed to be exponentially distributed with $\lambda = 0.08d$ where d is the true distance between the mobile station and corresponding reference point [11]. Because in our environment the longest NLOS distance between mobile station and reference point is 500 m, we assumed $\lambda = 40$ m. The measurement error in power is assumed to be white random variable with zero-mean and with standard deviations of 12 dB [12]. The noises are assumed to be mutually independent.

The coordinates of the reference points are assumed to be R_1(0, 0), R_2(-300,300), R_3(300,300), R_4(300,-300) and R_5(-300,-300) [7]. Since there are three reference points to detect the signal while it is moving, other two reference points are not included to the simulations. The transmission parameters of the simulation system is selected according to [13] as uplink band of 830MHz and subcarrier spacing of 15KHz. Frequency band choice is done according to Walfisch – Ikegami Model's efficient frequency band interval and not any other requirements are sought. 1000 Monte Carlo runs are made to obtain the system's behavior.

The position estimates of single carrier location estimation system are shown in Fig. 3. Those of 32 carrier case are given in Fig. 4. The correlation of x and y coordinates of single carrier and multi-carrier cases are $\rho_{N=1} = -0.3$ and $\rho_{N=1} = -0.15$, respectively.

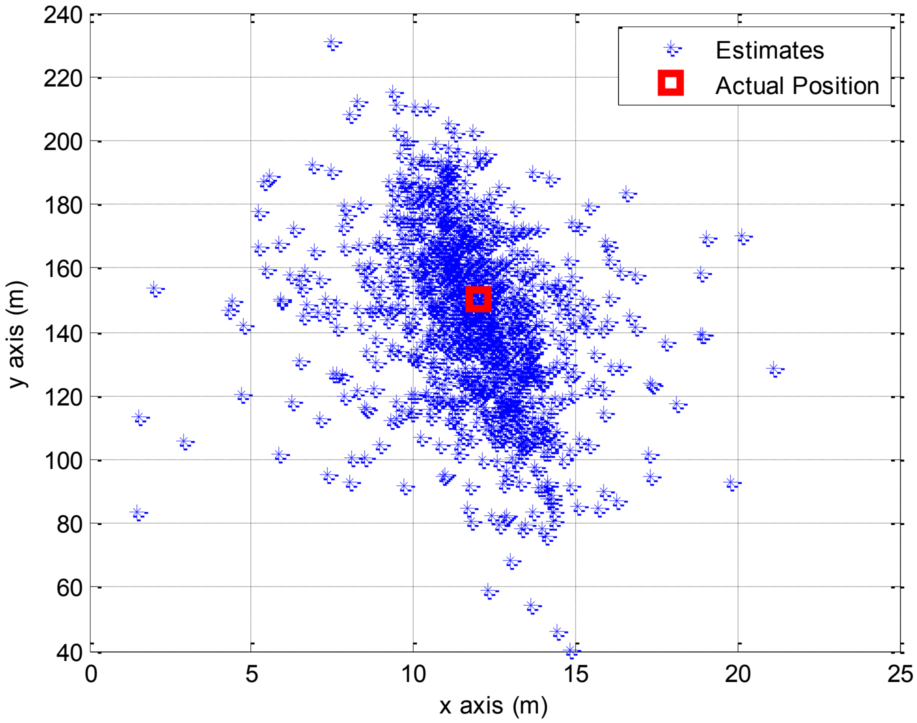

Figure 3. Position estimates of a single carrier system.

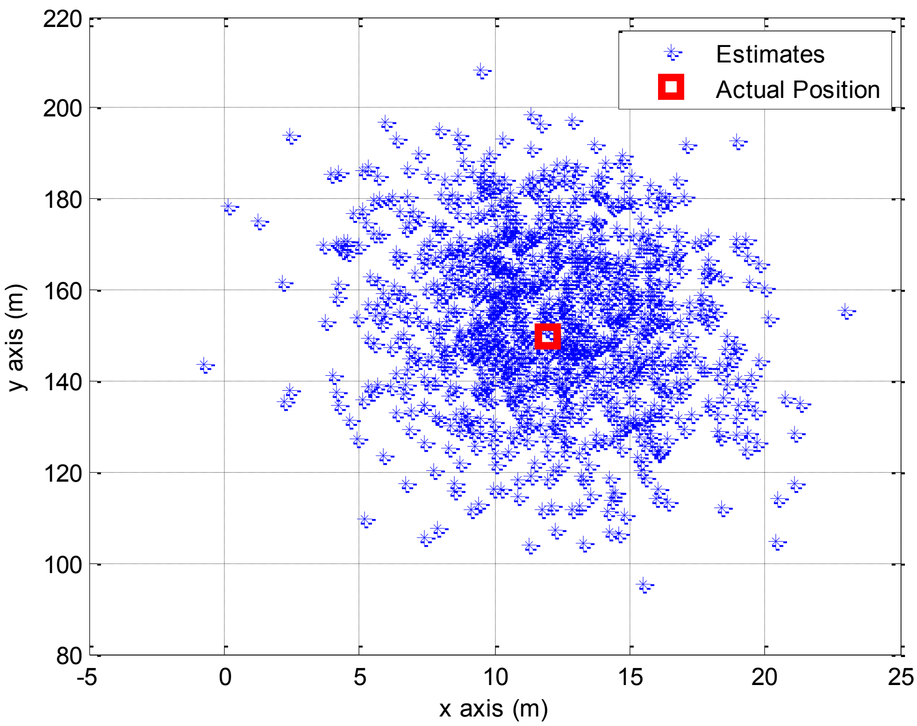

Figure 4. Position estimates of a multi-carrier system that makes use of multiple measurements (N=32).

The mean estimation error values in x and y directions of the single carrier system are 11.7 and 144, respectively. For the 32 subcarrier system these values are 11.9 and 151, respectively. Variances of the estimation errors are $\sigma^2_{x,N=1} \approx 4.5$, $\sigma^2_{y,N=1} \approx 715$ for the single carrier case and $\sigma^2_{x,N=1} \approx 1.27$, $\sigma^2_{y,N=32} \approx 348$ for the multi-carrier case after 1000 Monte Carlo runs. These results also demonstrate that the accuracy of the location estimation systems can be improved when using multi-carrier modulation schemes. This analysis is also repeated for several distinct position values and same results are observed.

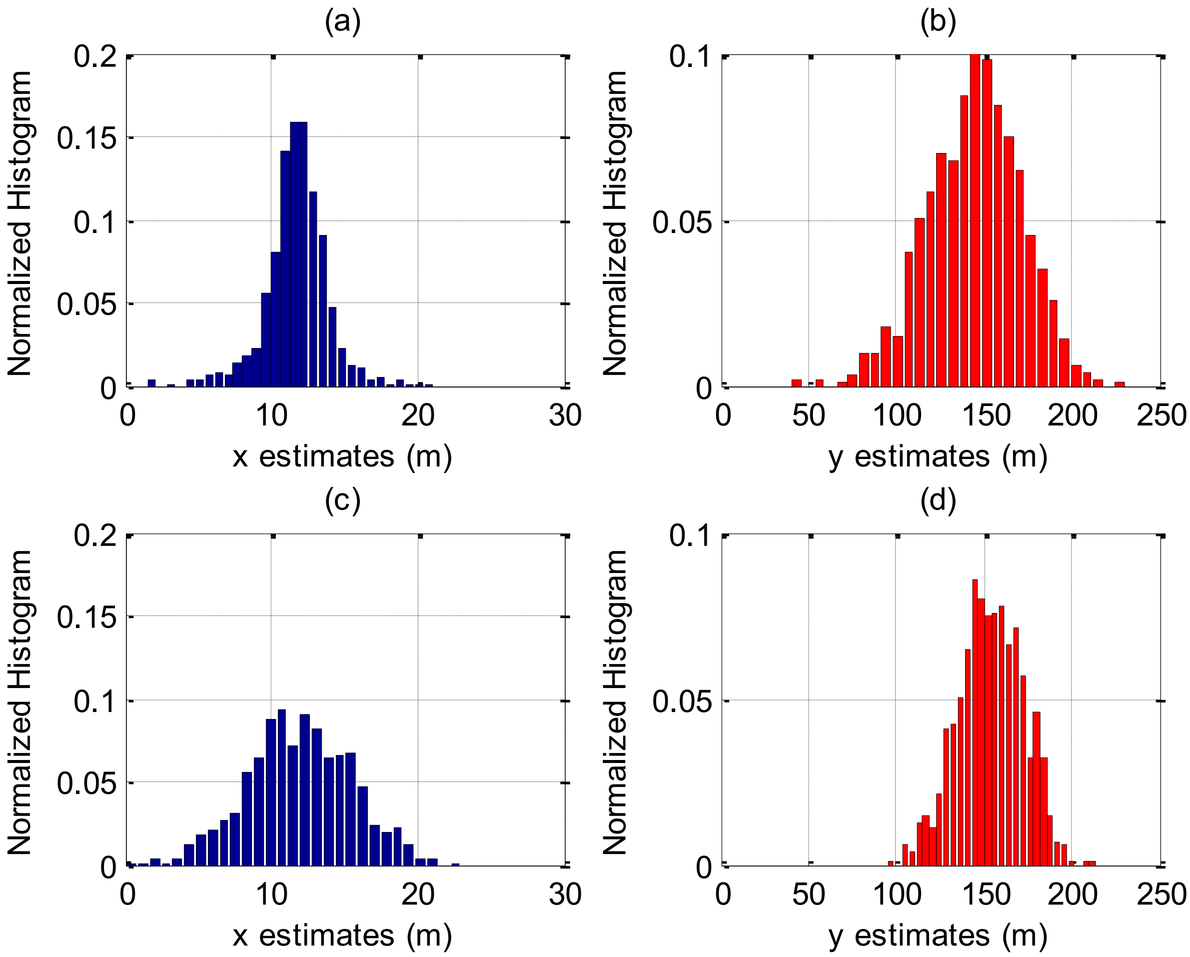

Figure 5. Corresponding histograms of x and y estimates:

(a) x estimates of single carrier system

(b) y estimates of single carrier system

(c) x estimates of multi-carrier system that makes use of multiple measurements (N=32)

(d) y estimates of multi-carrier system that makes use of multiple measurements (N=32).

Histograms of the position estimates in x and y directions of measurement and multiple measurements are shown in Fig.5. The position estimates in x and y directions of the multi-carrier system with N=32 can be considered Gaussian according to the Jarque–Bera test is a goodness-of-fit measure of departure from normality at the 5 % significance level. The estimates of the single carrier system fail this test at the same significance level, hence they can not be considered as Gaussian random variables. This result justifies the expression in (1), which is stated based on the central limit theorem.

The histograms of the radial estimation errors of the considered configurations are shown in Fig. 6. Corresponding scaled pdf in (3) integrated over the interval $[0, 2\pi)$ of θ is also shown in Fig. 6 (b). As can be observed from the figure, the pdf provides a good match with the simulation results.

Based on the analysis of the radial estimation error performance levels of location based services can easily be obtained. Furthermore, results presented here can be used to demonstrate the compliance of the location estimation system performance with the limits that are determined by the regulatory organizations. Based on the cdf of the estimation error for 32 measurements, the FCC requirements and the compliance levels are given in Table 1. From the table it can be observed that both systems satisfy the requirements for the fixed position. However, the estimation accuracy of the multi-carrier system is higher due to the data fusion.

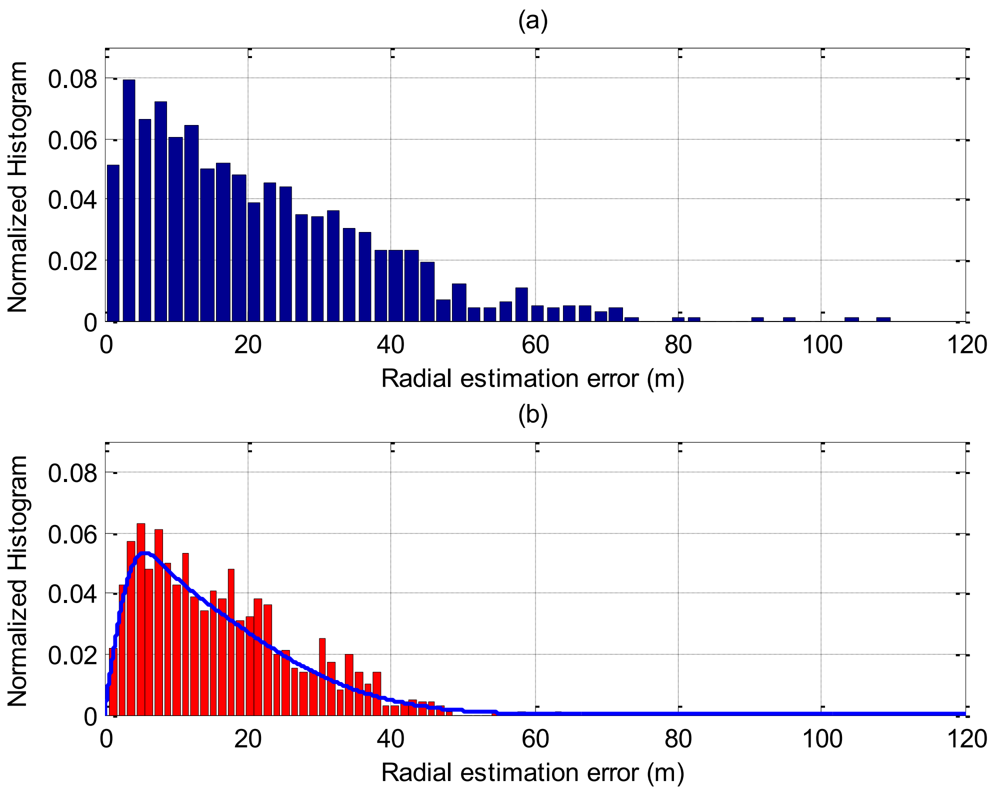

Figure 6. Corresponding histograms of radial estimation error estimates:

(a)Single measurement

(b) Multiple measurements (N=32) and the corresponding analytical pdf expression.

The analytical expression of the cdf of for N=32 system is shown in Fig. 7. The cdf of the simulated estimates are also given in Fig. 7. From this figure we can observe that there is a good match between simulation and analytical results.

TABLE I. FCC REQUIREMENTS [1]

FCC Requirement Estimation Error	FCC 67 %	FCC 95%
	100m	*300m*
N=1	24.3m	53.8m
N=32	18.5m	36.7m

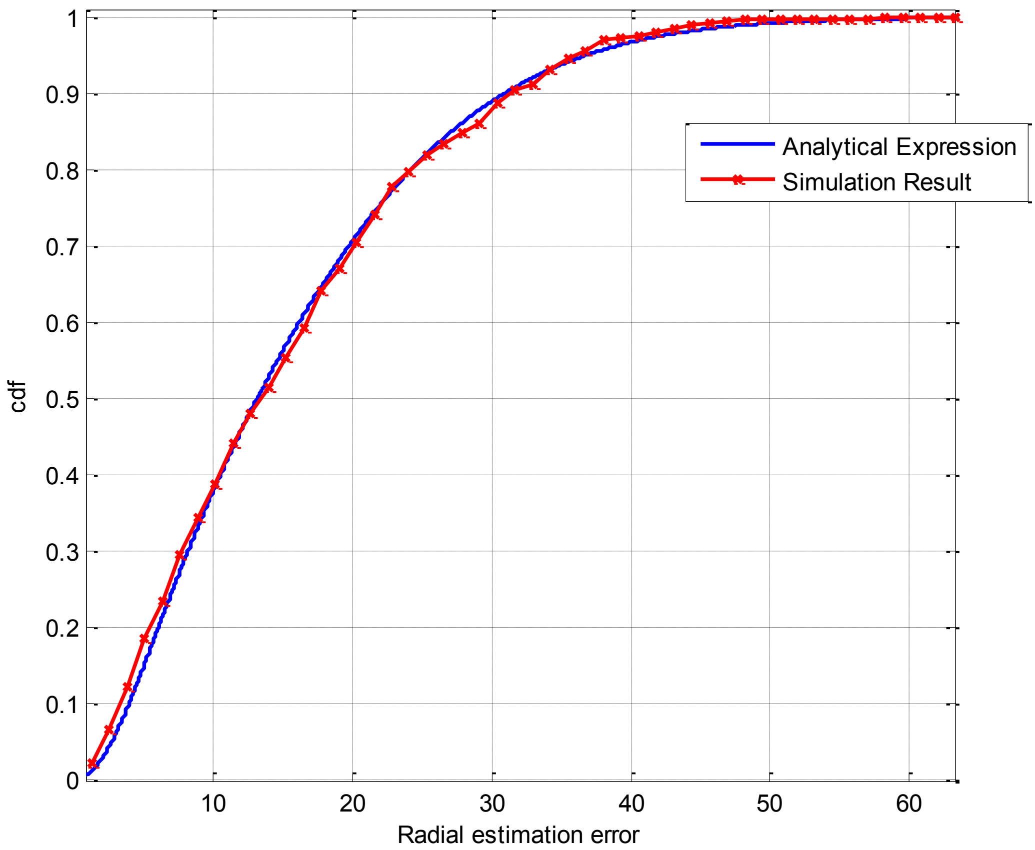

Figure 7. Comparison of the analytical and simulation results for N=32.

V. CONCLUSION

Novel communication standards making use of multi-carrier multi-antenna systems, lead to location estimation systems with multiple measurements. In this report, we have shown that when using data fusion for location estimation in these systems, the central limit theorem can be used to provide a good approximation to understand the behavior of coordinate estimates.

We have shown that via simulation results that the position estimates of the multi-carrier system in x and y directions can be considered Gaussian according to the Jarque–Bera test is a goodness-of-fit measure even when only 32 carriers are used. Based on this approximation, the probability density function of the radial estimation error is given. By making use of this expression, regulatory bodies and mobile network operators can easily estimate the location estimation accuracy of the systems in use.

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