

# *Signal processing in the upcoming wireless networks:*

*Untangling signals in space, in spectrum, and network coded*

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Department of Information Science and Technology



instituto de  
telecomunicações

*creating and sharing knowledge for telecommunications*

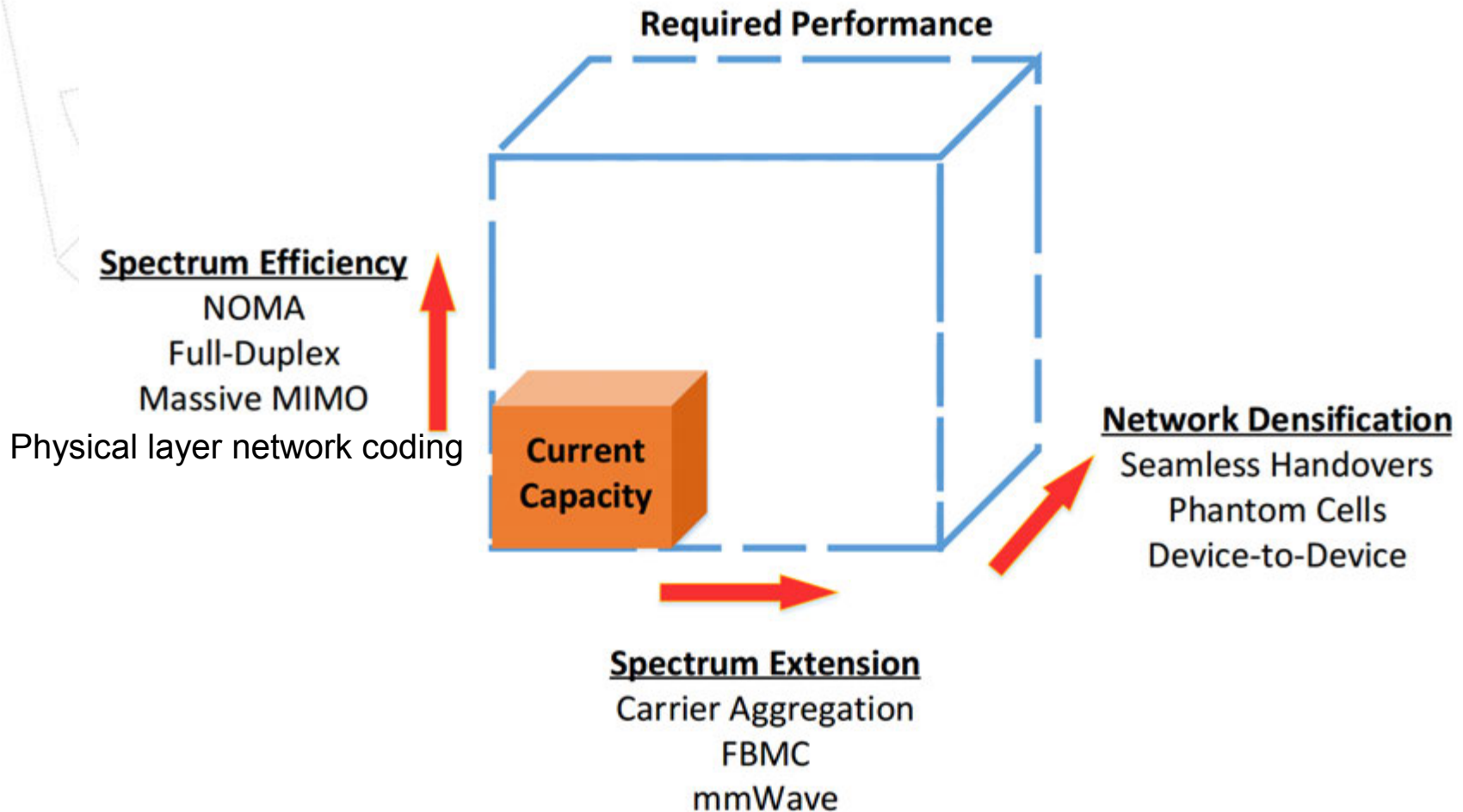
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**DARNEC'15**

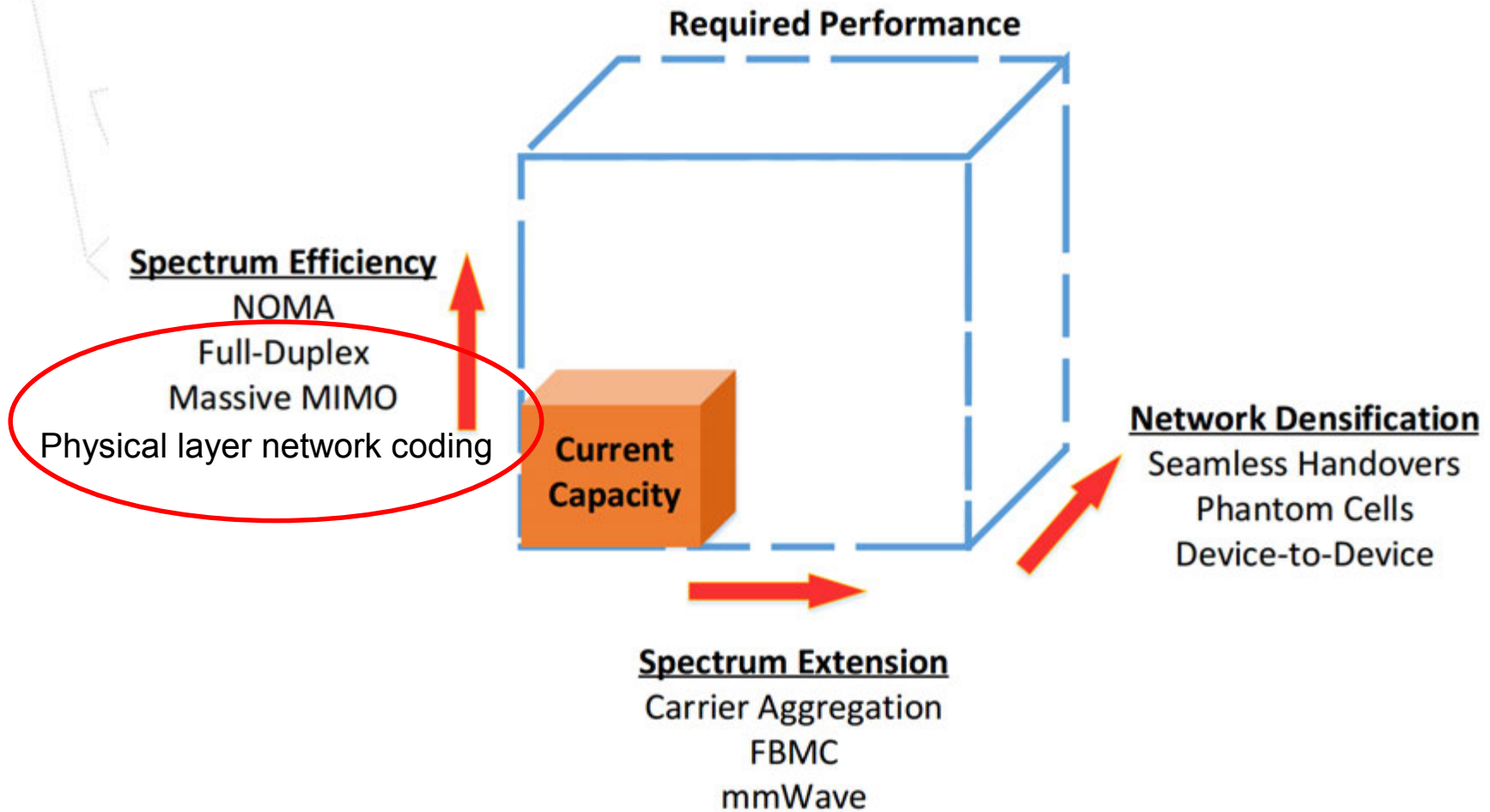
**4<sup>th</sup> Nov. 2015**

**Istanbul**

# How to offer more?



# How to offer more?



# *Signal processing in the upcoming wireless networks:*

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- 1- Spatial multiplexing (MIMO)**
- 2- Massive MIMO**
- 3- In-band full-duplex**
- 4- TWRC and the Y-network**
- 5- Full-duplex + massive MIMO + PLNC**

***Signal processing in the  
upcoming wireless networks:  
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**1- Spatial multiplexing (MIMO)**

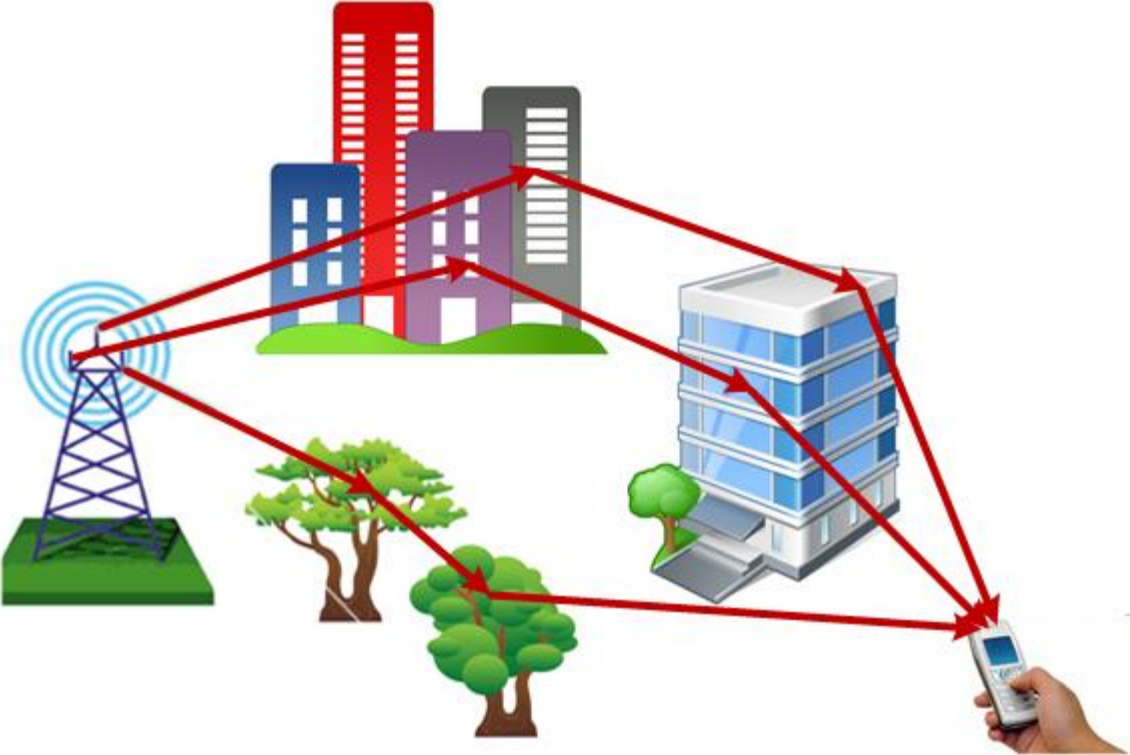
**2- Massive MIMO**

**3- In-band full-duplex**

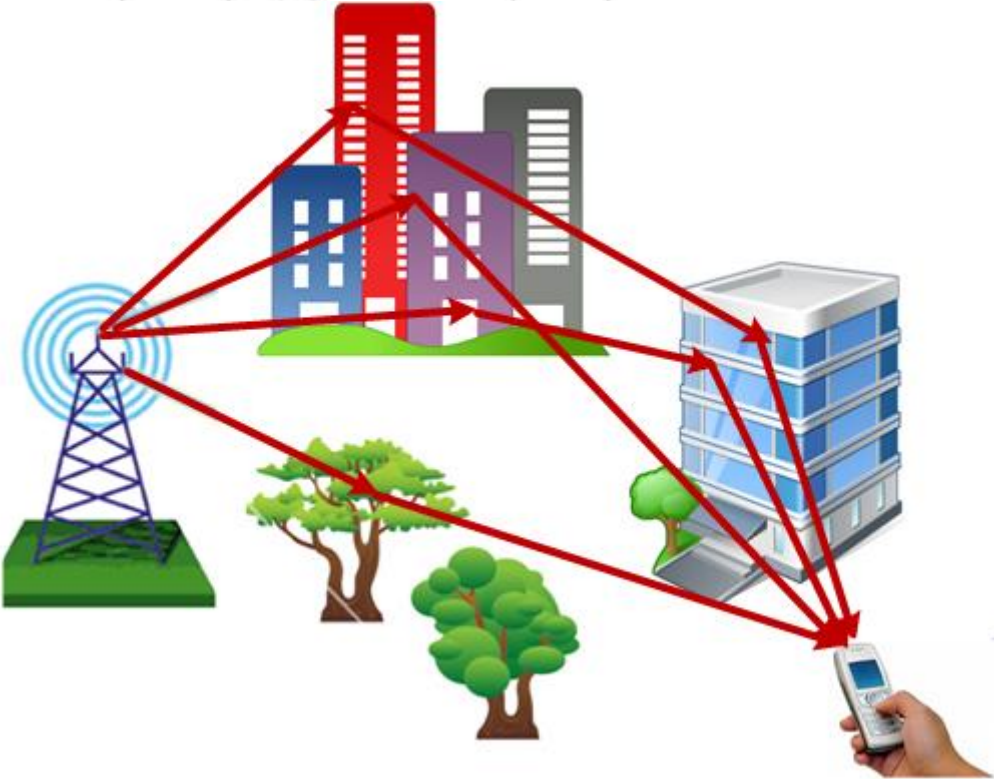
**4- TWRC and the Y-network**

**5- Full-duplex + massive MIMO + PLNC**

# Multipath Fading

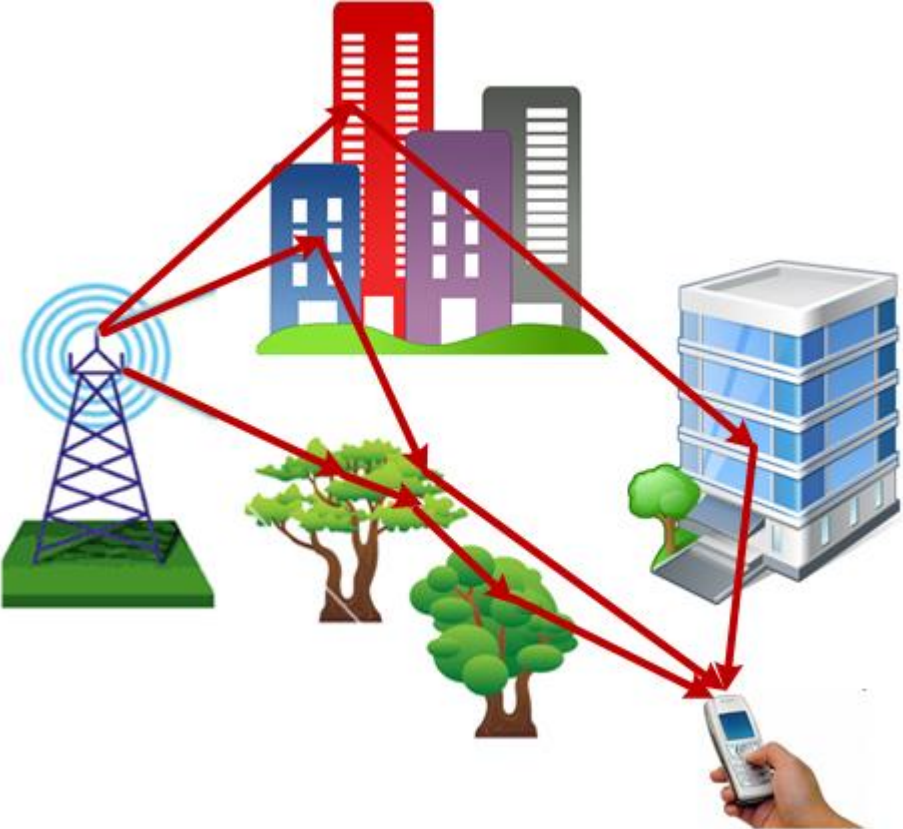


# Multipath Fading



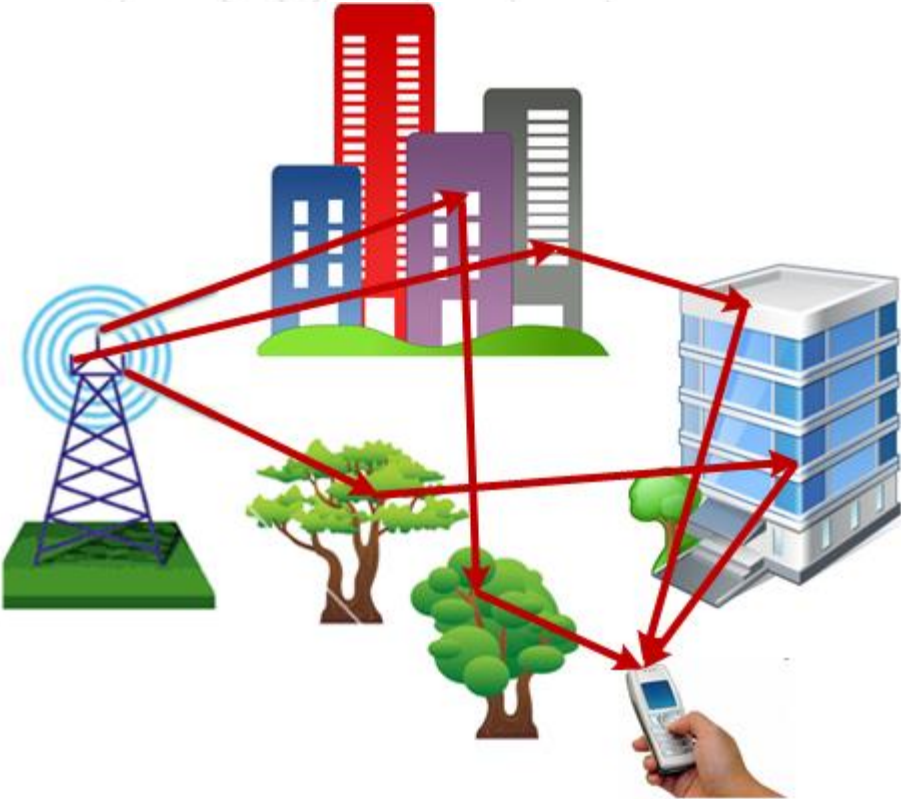


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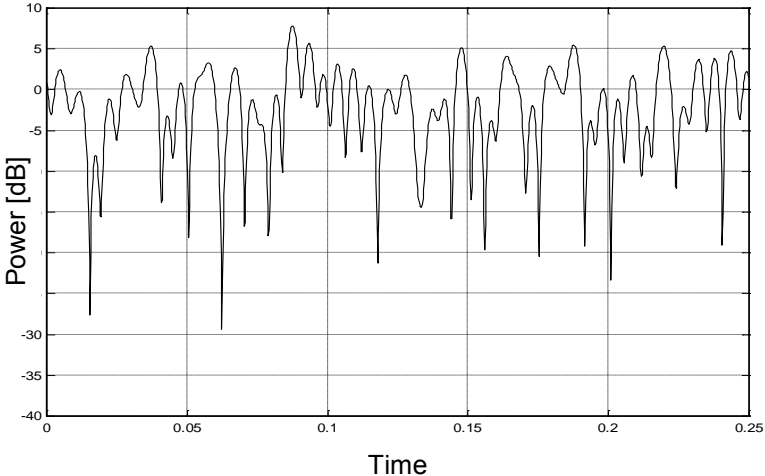
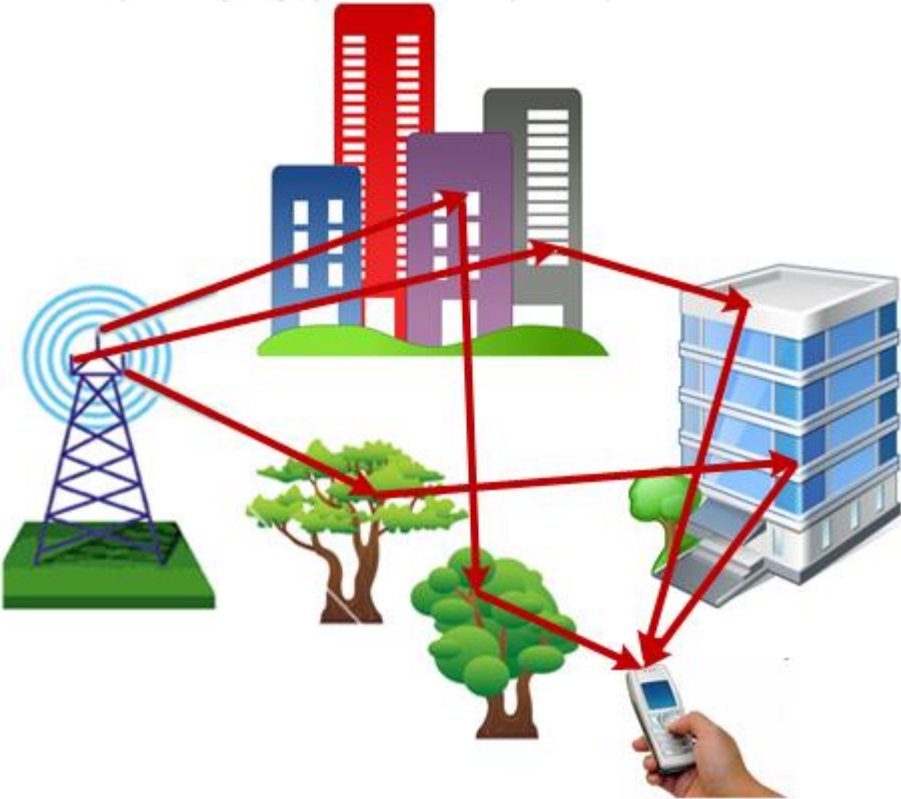




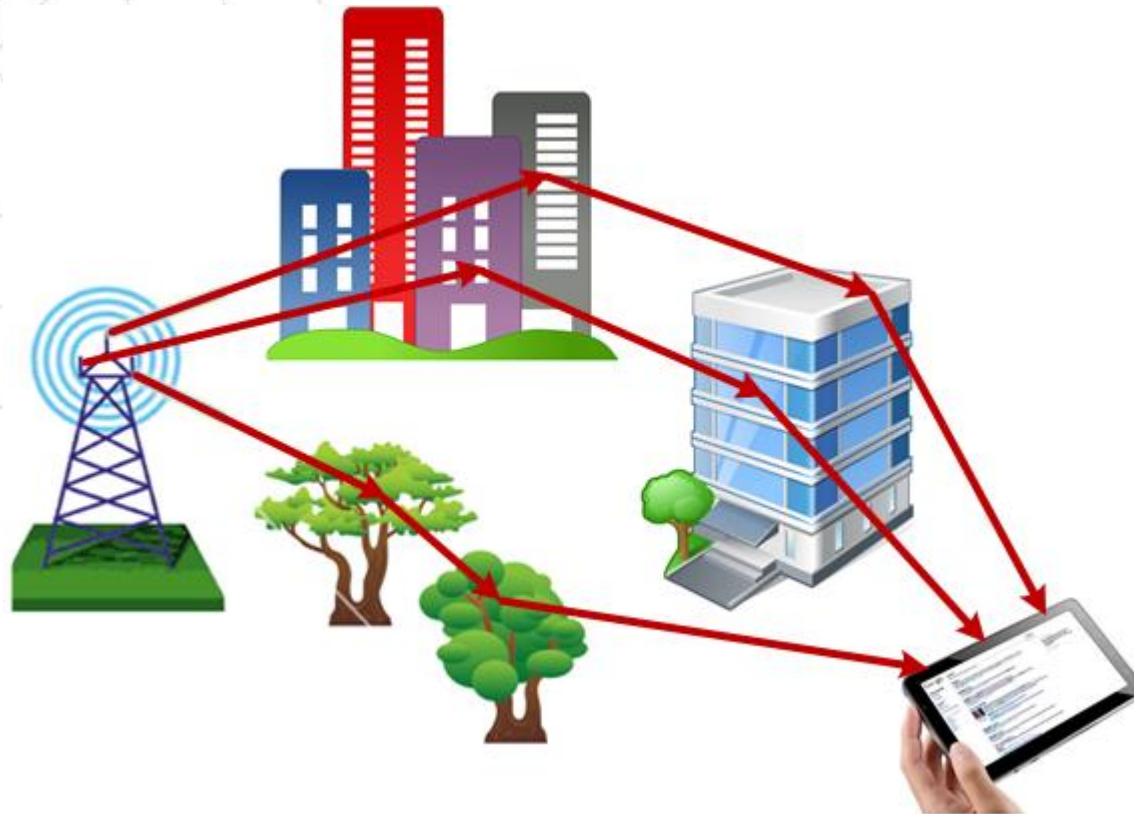
# Multipath Fading



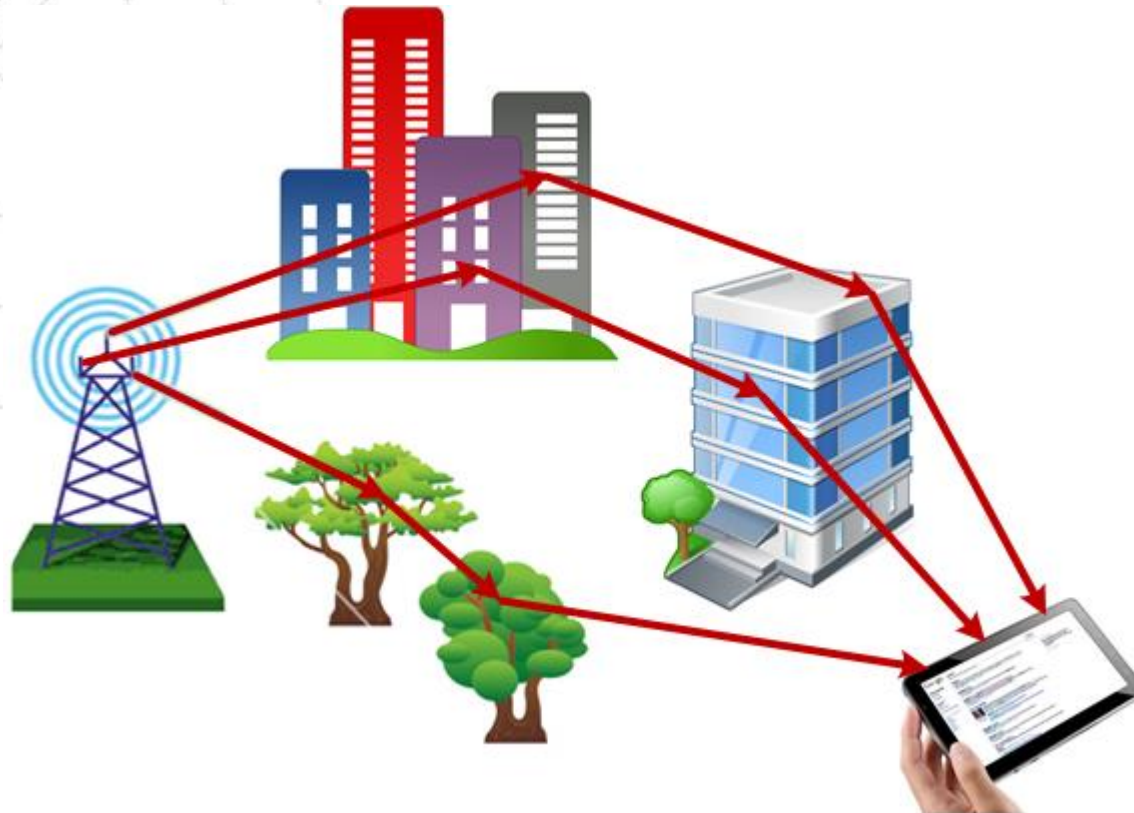
# Multipath Fading



It turns out that this can be made equivalent to...



It turns out that this can be made equivalent to...



How?

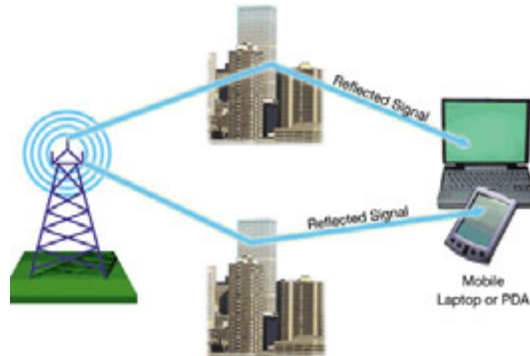
# Beyond Shannon's capacity

1993 - 1998:

- Paulraj et al. research on MIMO (Stanford, CA).
- Gerard Foschini deduced the theoretical capacity for MIMO and proved it experimentally (@ Bell Labs, NJ).



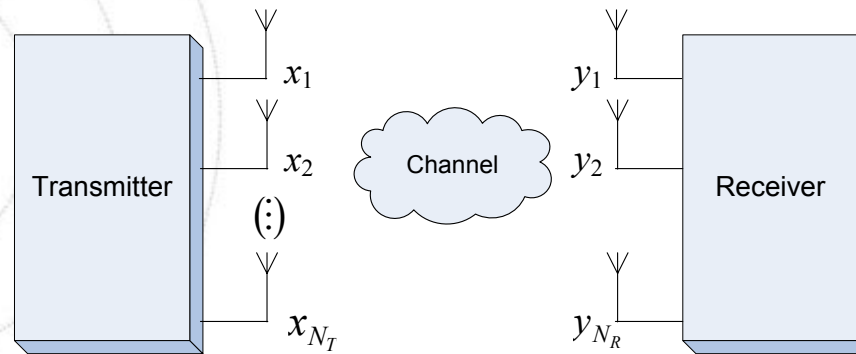
**Gerard Foschini**  
(Bell Labs, NJ)



- Multipath can be beneficial by opening simultaneous channels.
- Multiple-input multiple-output (MIMO) was born.
- After 15 years of academic research → standards:  
802.11n and 4G (LTE-A, WiMax).



# Multiple-input multiple-output (MIMO) detection



$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_{N_R} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N_T} \\ h_{21} & h_{22} & \cdots & h_{2N_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R 1} & h_{N_R 2} & \cdots & h_{N_R N_T} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_{N_T} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \vdots \\ \mathbf{n}_{N_R} \end{bmatrix}$$

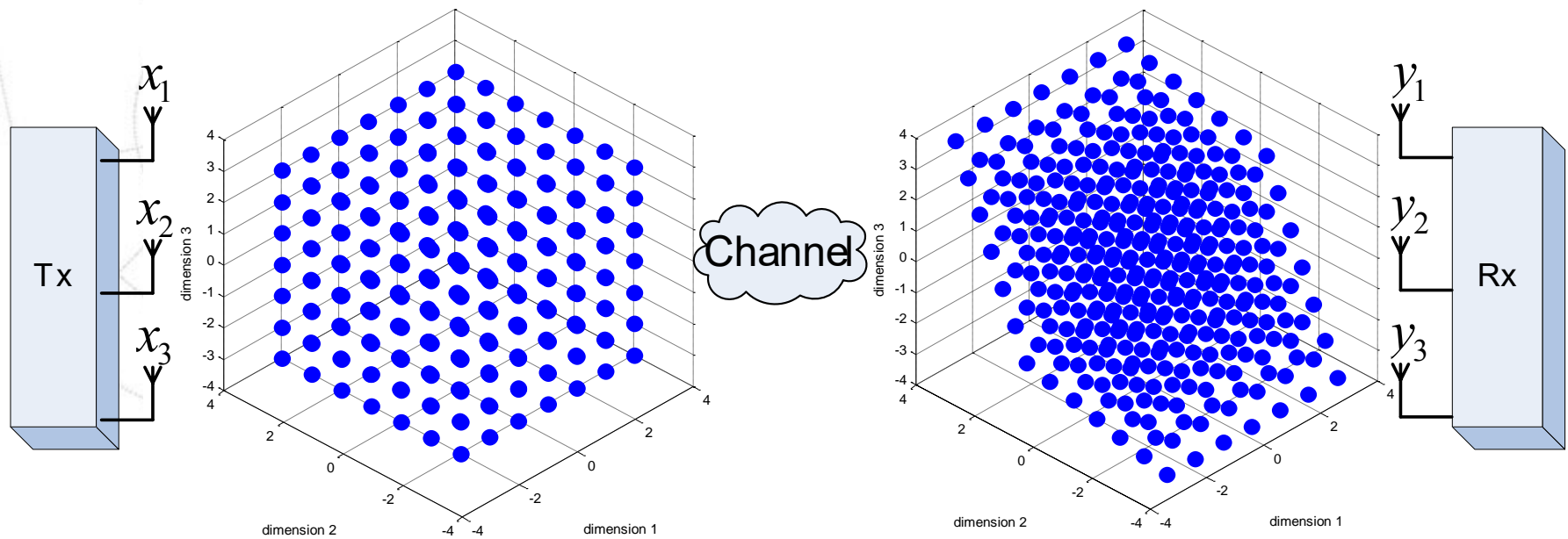
$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

$$\hat{\mathbf{x}}_{ML} = \min_{\mathbf{x}} \left\{ \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \right\}$$





# MIMO channel is a linear transformation

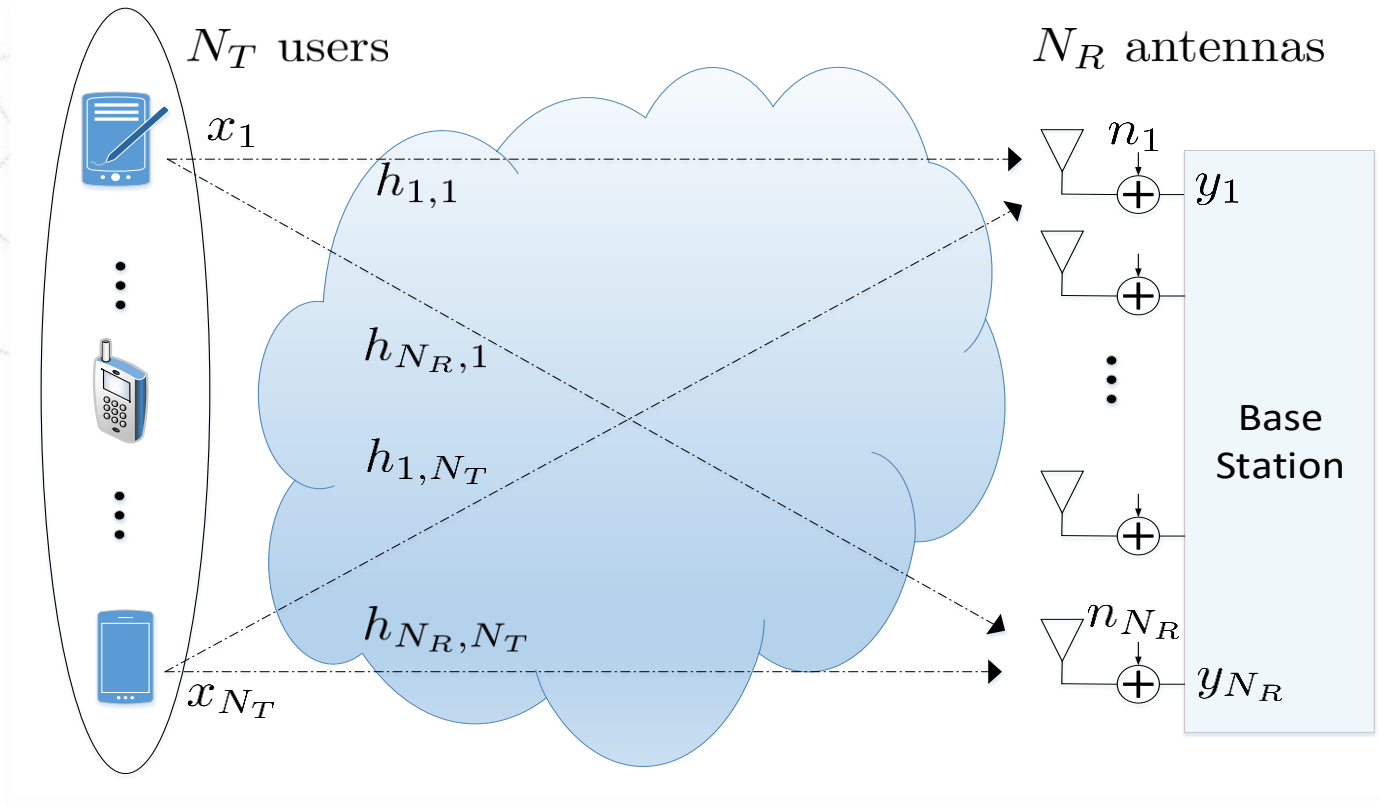


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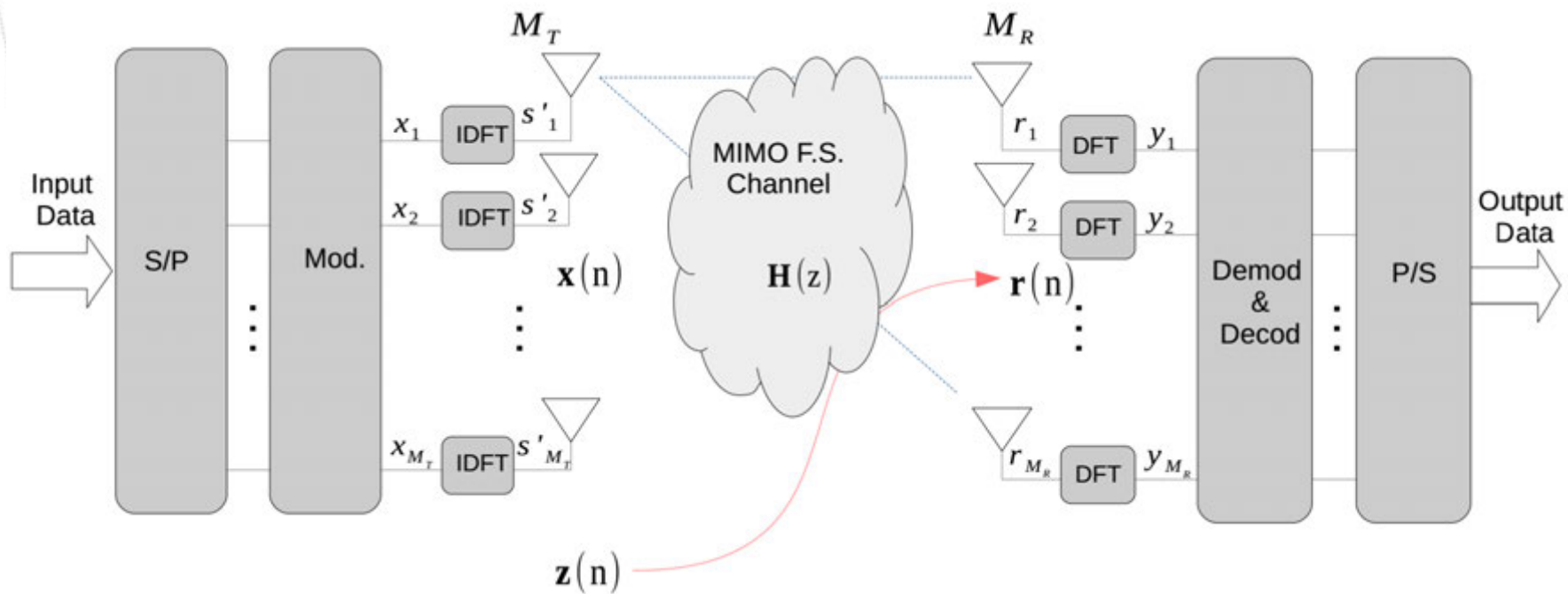
$$\hat{\mathbf{x}}_{ML} = \min_{\mathbf{x}} \left\{ \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \right\}$$

Example: 3 dimensions (3 antennas using PAM).

# Virtual MIMO (non co-located antennas)



# Orthogonal frequency division multiplexing MIMO (OFDM-MIMO)



# “A rose by any other name would smell as sweet”

	MIMO	Equalisation for ISI channels	Multi-user Communication
Inversion (linear)	<ul style="list-style-type: none"> <li>• Zero-forcing (ZF)</li> <li>• Channel inversion</li> <li>• Decorrelation</li> </ul>	Zero-forcing (ZF) equalisation	Decorrelating
Minimum mean squared error (MMSE)	MMSE	MMSE filtering	MMSE detection
Interference cancellation	<ul style="list-style-type: none"> <li>• Nulling and cancelling</li> <li>• Successive interference cancellation (SIC)</li> <li>• V-BLAST detection</li> </ul>	Decision feedback equalisation (DFE)	<ul style="list-style-type: none"> <li>• Iterative multi-user detection MUD)</li> <li>• Successive interference cancellation (SIC)</li> </ul>
Optimum detection	<ul style="list-style-type: none"> <li>• Maximum likelihood detection (MLD)</li> <li>• Exhaustive search</li> </ul>	Maximum likelihood sequence detection (MLSD)	<ul style="list-style-type: none"> <li>• ML detection</li> <li>• Brute force</li> <li>• Sphere decoding (near optimum)</li> </ul>
Precoding	<ul style="list-style-type: none"> <li>• Multiuser-MIMO</li> <li>• Broadcast channel (BC)</li> </ul>	<ul style="list-style-type: none"> <li>• ISI Precoding</li> <li>• Costas precoding</li> <li>• Tomlinson-Harashima precoding (THP)</li> </ul>	Dirty paper coding (DPC)
Parallel subchannels	<ul style="list-style-type: none"> <li>• Closed loop SU-MIMO</li> <li>• Singular value decomposition (SVD) and water filling</li> <li>• Communication over eigen- modes</li> <li>• Eigen-beam spatial division multiplexing</li> <li>• Precoding</li> <li>• Beamforming</li> </ul>	<ul style="list-style-type: none"> <li>• OFDM</li> <li>• Multi-tone modulation</li> <li>• Filter bank multicarrier</li> </ul>	Not defined

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**2- Massive MIMO**

**3- In-band full-duplex**

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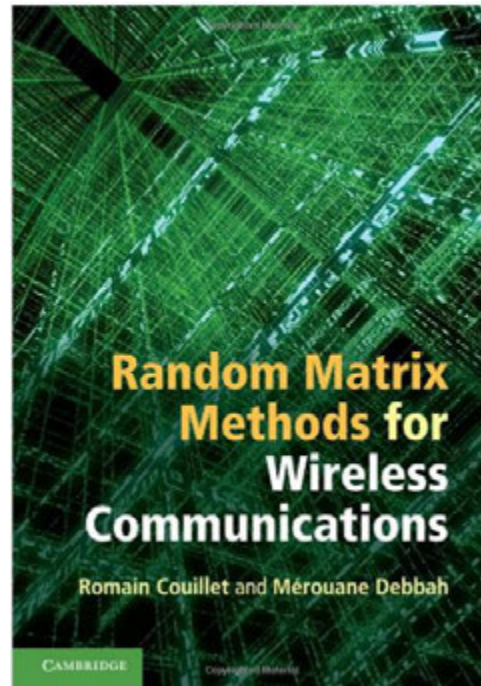
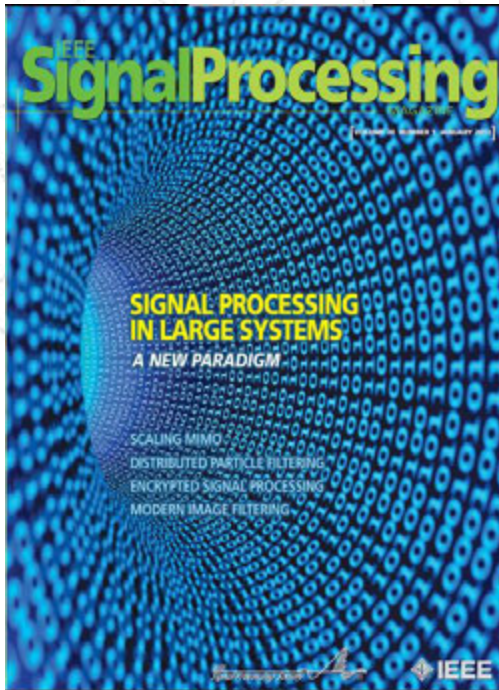
**3- In-band full-duplex**

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# Massive MIMO



F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, “Scaling up MIMO: Opportunities and Challenges with Very Large Arrays”, IEEE Signal Processing Magazine, **January, 2013**.

# Efficient Linear Processing (for tall H matrices)

- Zero-forcing (or MMSE):

$$\underbrace{(\mathbf{H}^H \mathbf{H})}_{\mathbf{Z}}^{-1} \mathbf{H}^H \mathbf{y} = \mathbf{H}^\dagger \mathbf{y}$$

- Near optimal diversity:

$$N_R - N_T + 1 \approx N_R$$

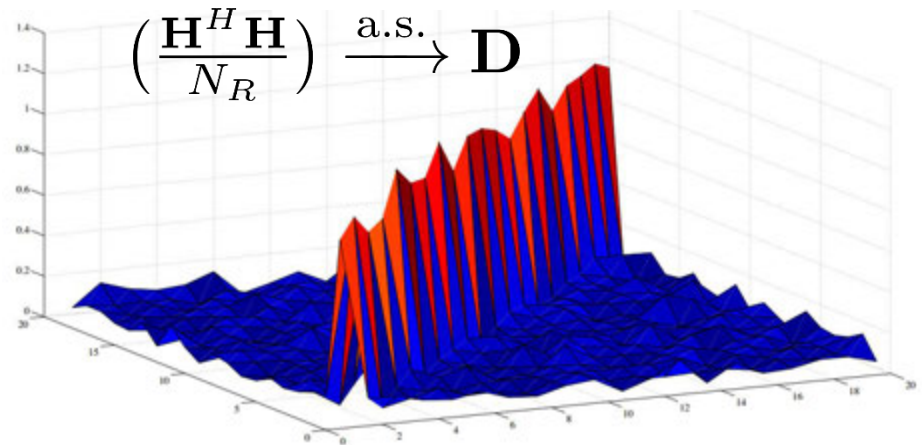
- Low complexity

- But requires inverse ( $\mathcal{O}(N_T^3)$ )

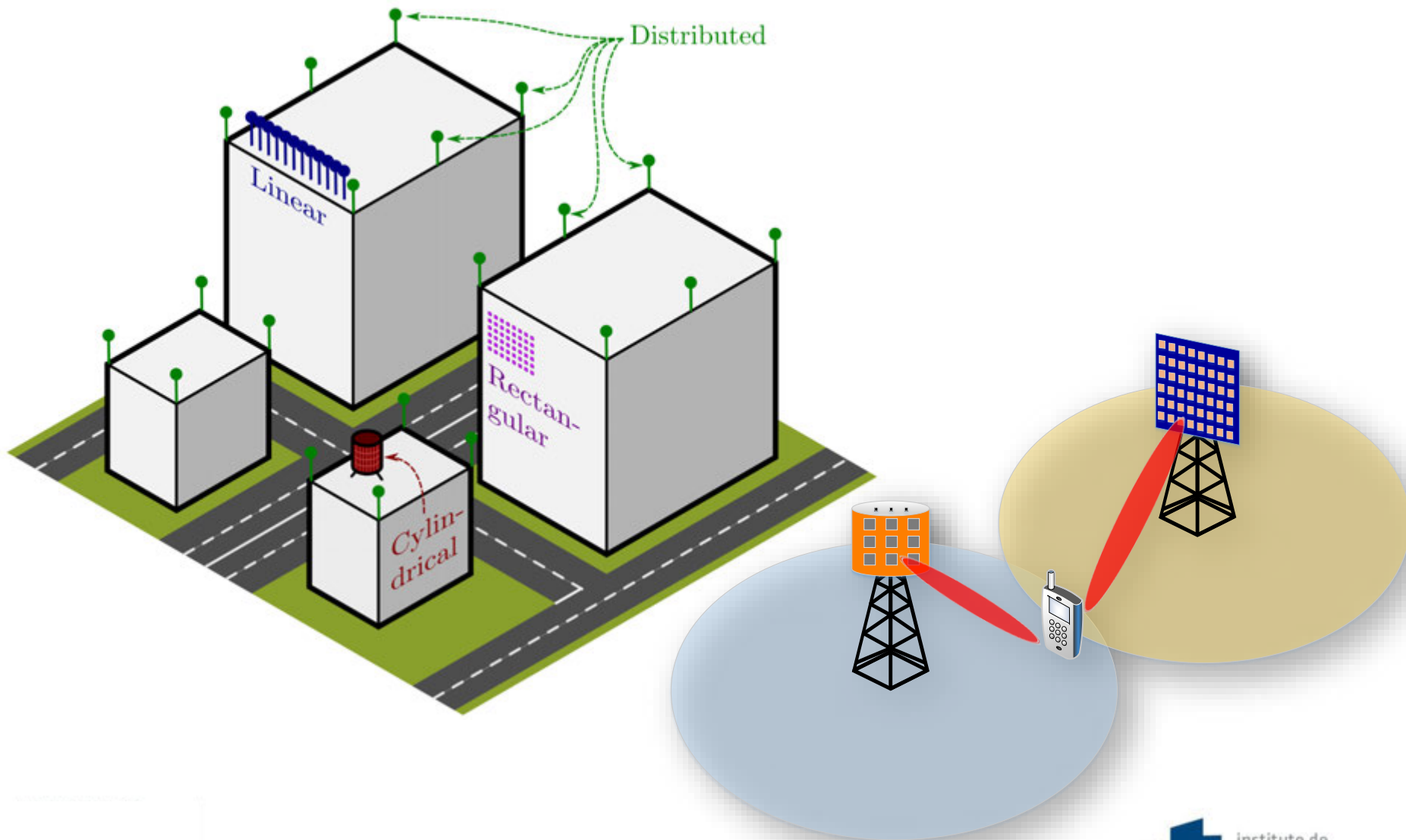
- It is possible to use:

Neumann series (approximation)

Matrix inversion lemma

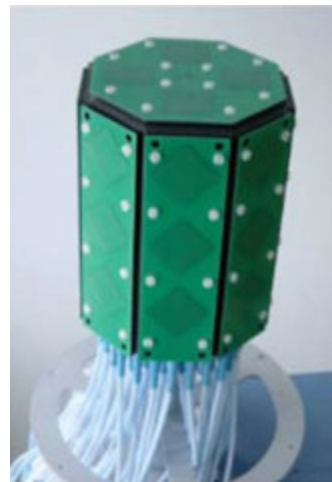
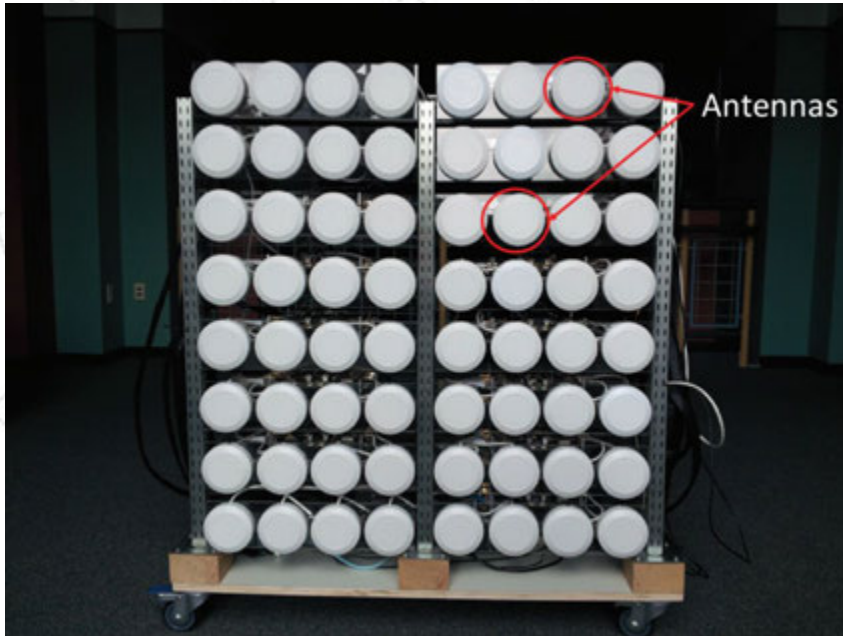


# Massive MIMO: antenna arrays





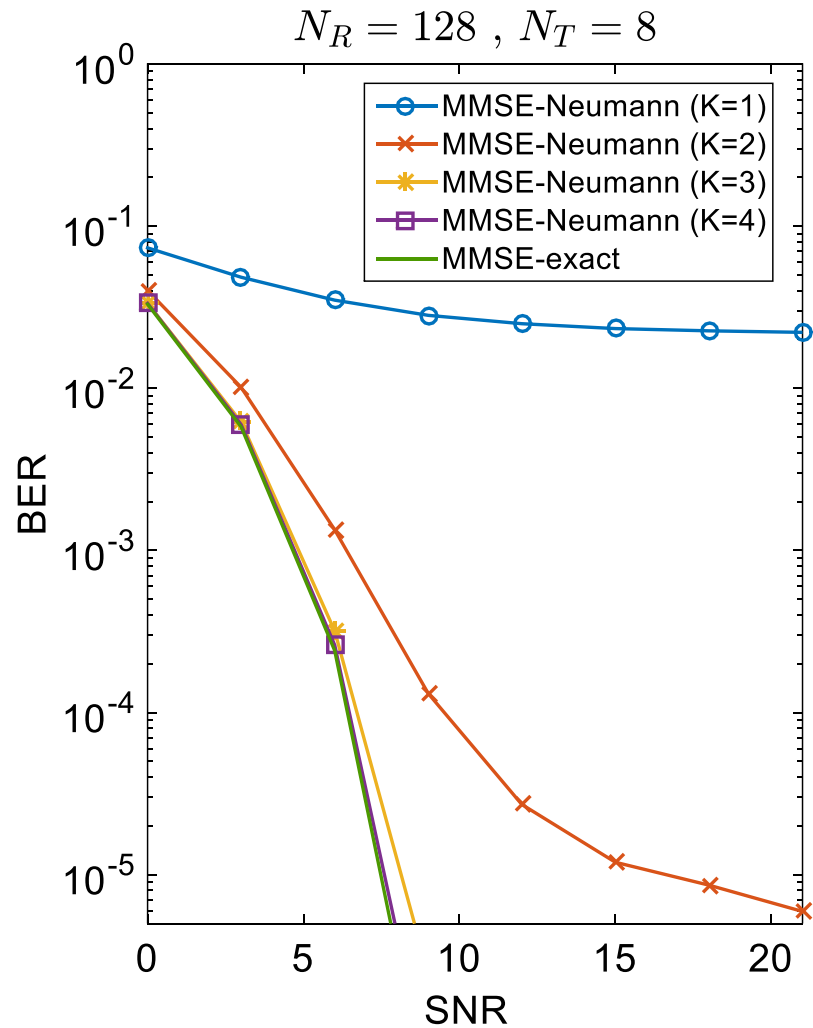
# Massive MIMO: antenna arrays



(Images: from Lund Univ., EU METIS and ARGOS projects)

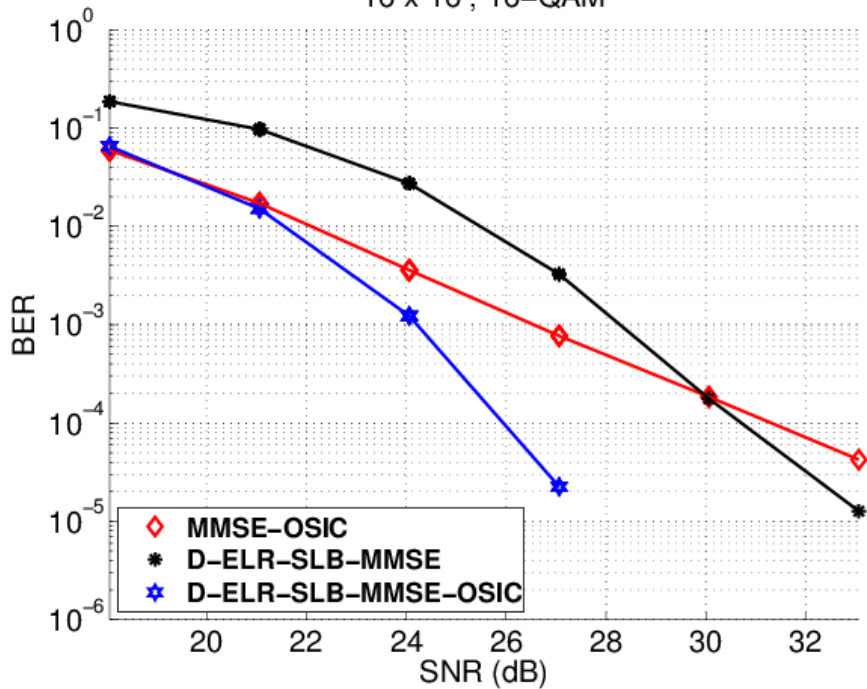
# Neumann series (approximation)

The Neumann series provides an efficient, hardware-realisable, method to compute the inverse required to perform linear processing.

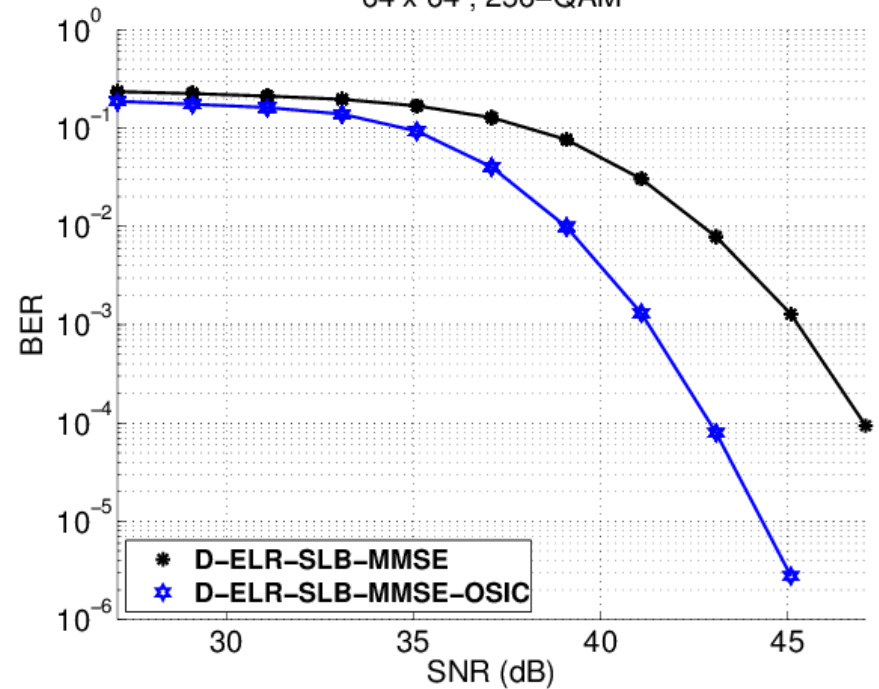


# Large (symmetric) MIMO

16 x 16 , 16-QAM



64 x 64 , 256-QAM





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## Recent state-of-the-art in radio science

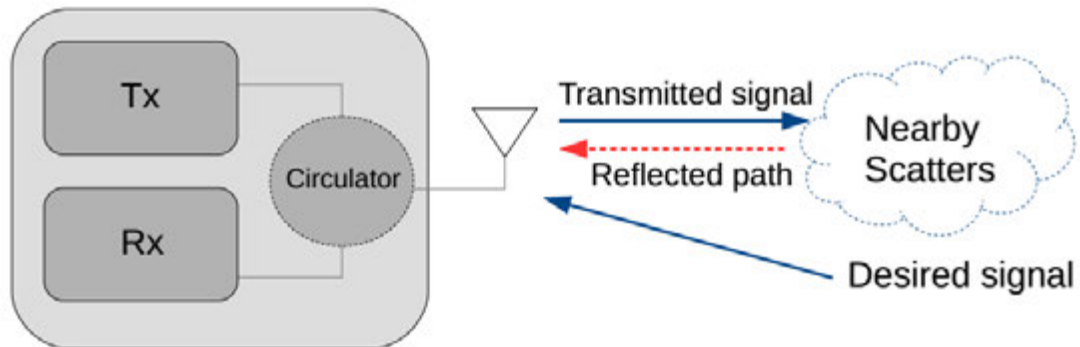
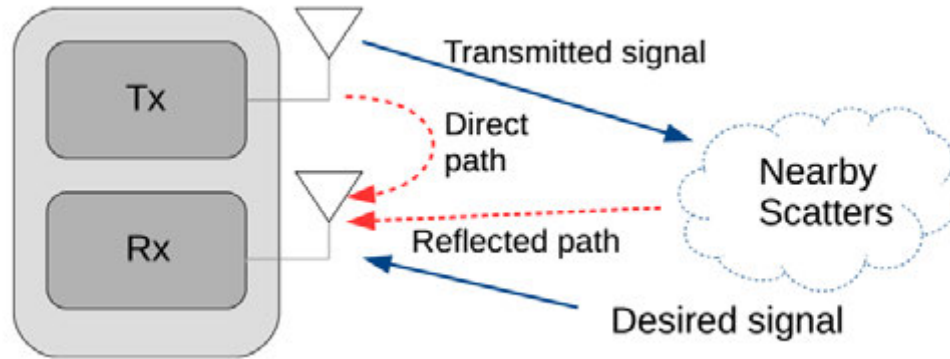
*“It is generally not possible for radios to receive and transmit on the same frequency band because of the interference that results.”*



Andrea Goldsmith,

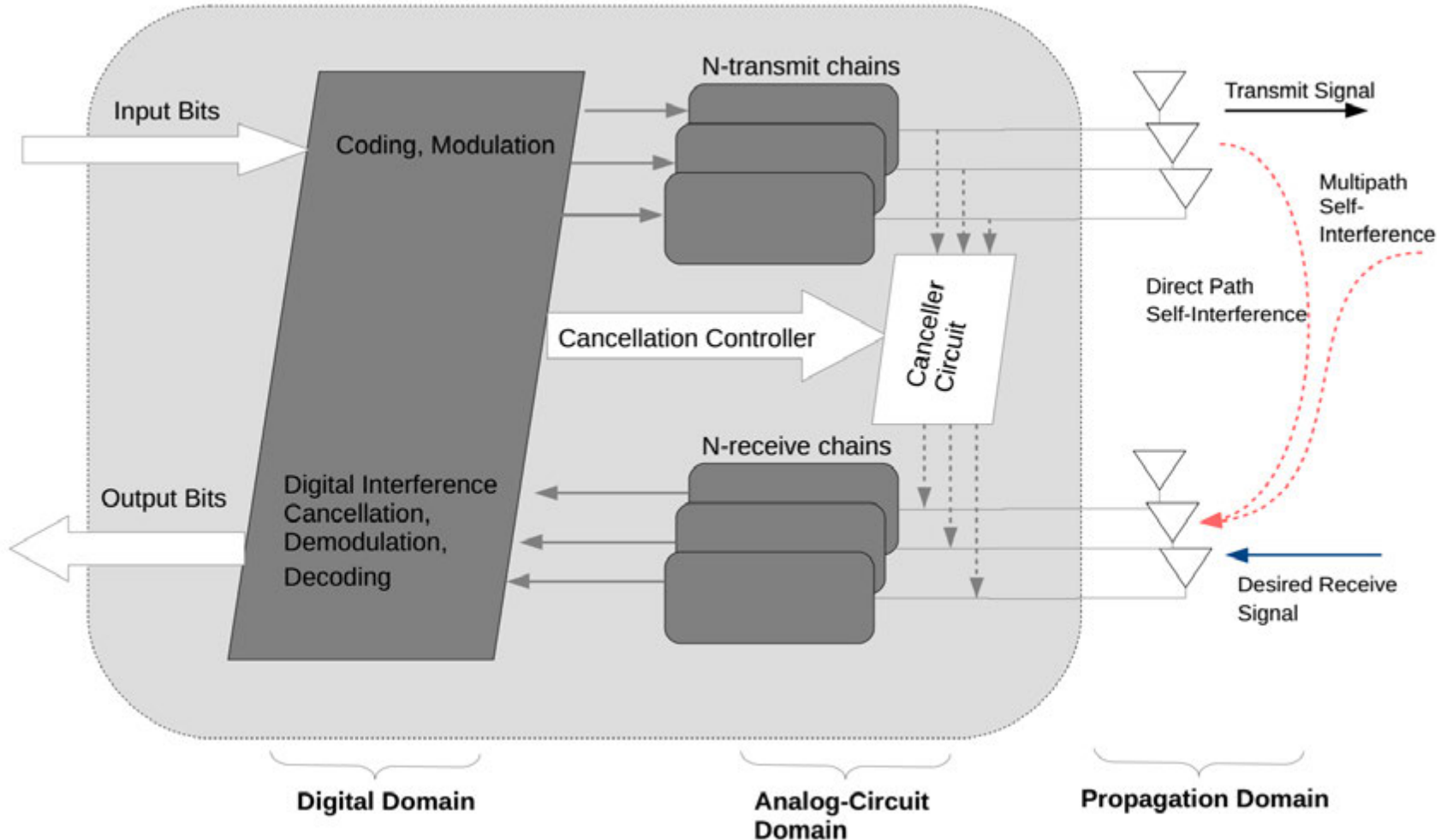
In *Wireless Communications*, Cambridge University Press, p. 454, 2003

# Two possible antenna setups

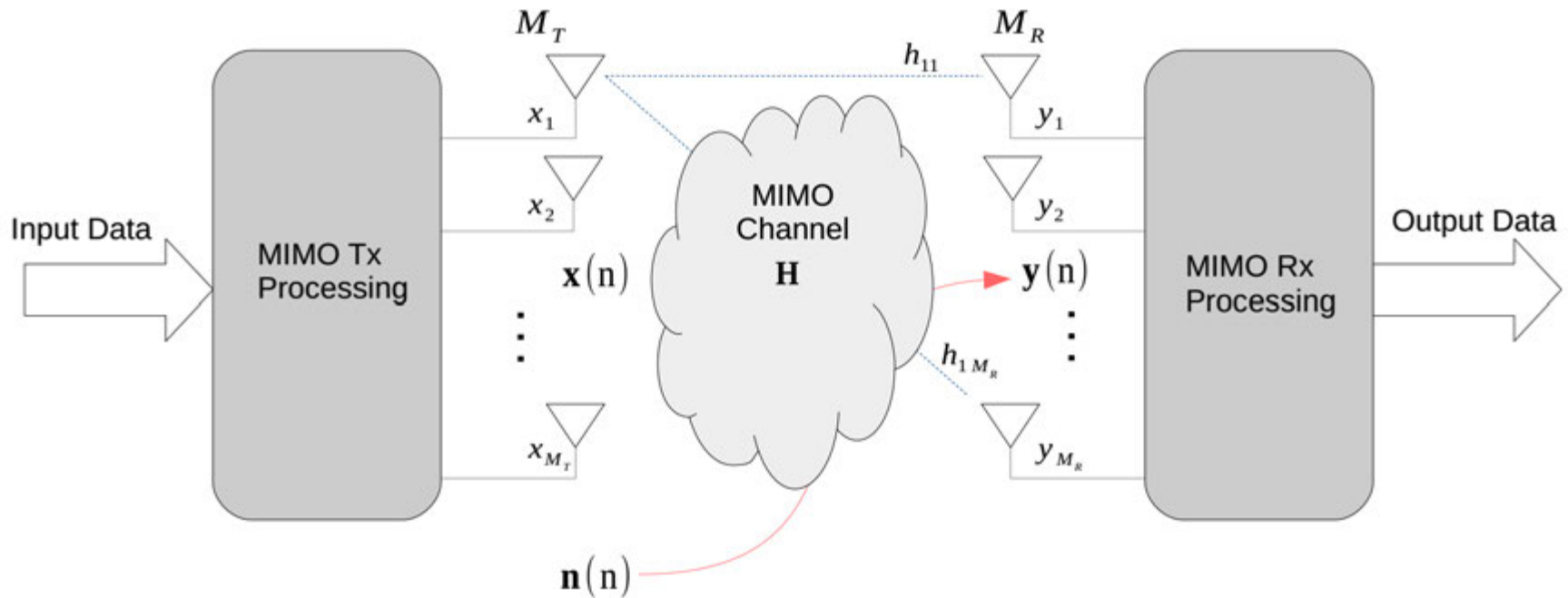


# MIMO full-duplex device

## In-Band Full-Duplex Terminal

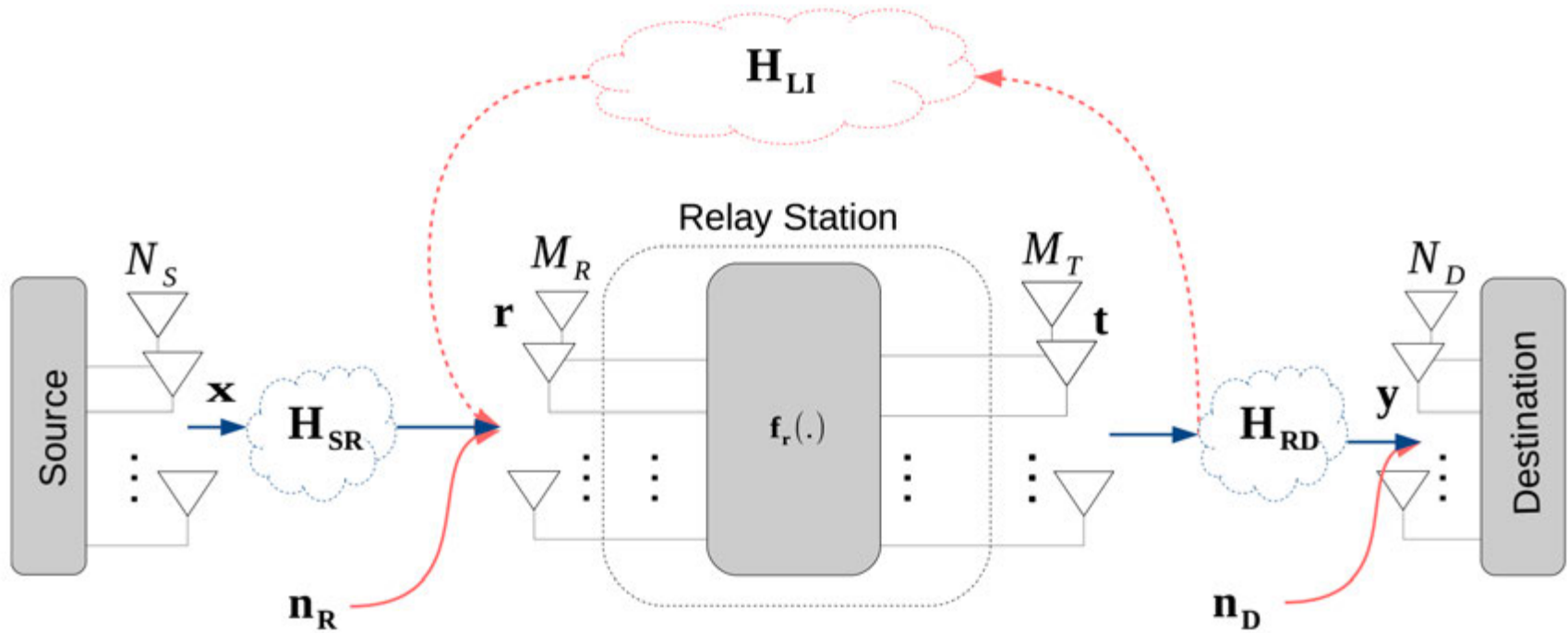


# Multiple-input multiple-output (i.e., multi-antenna)

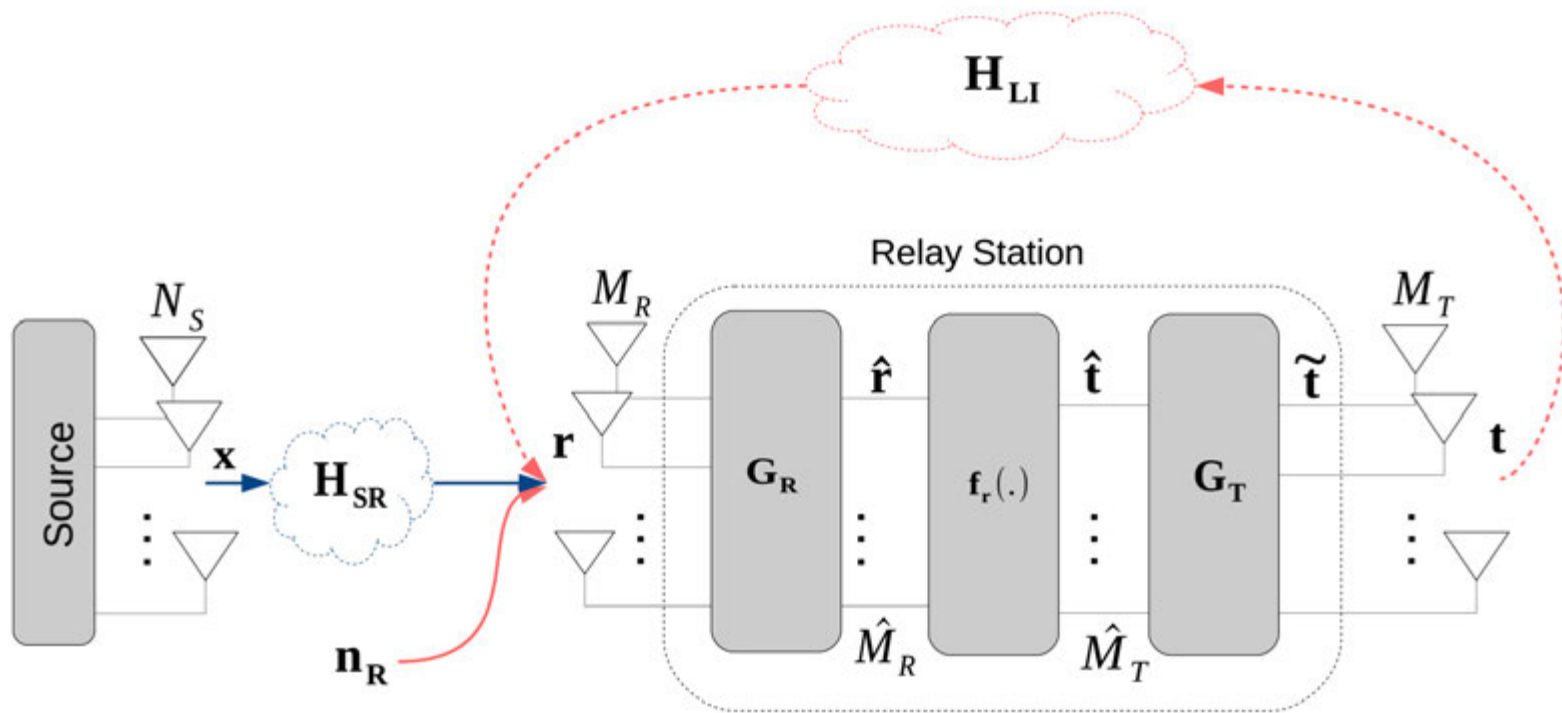




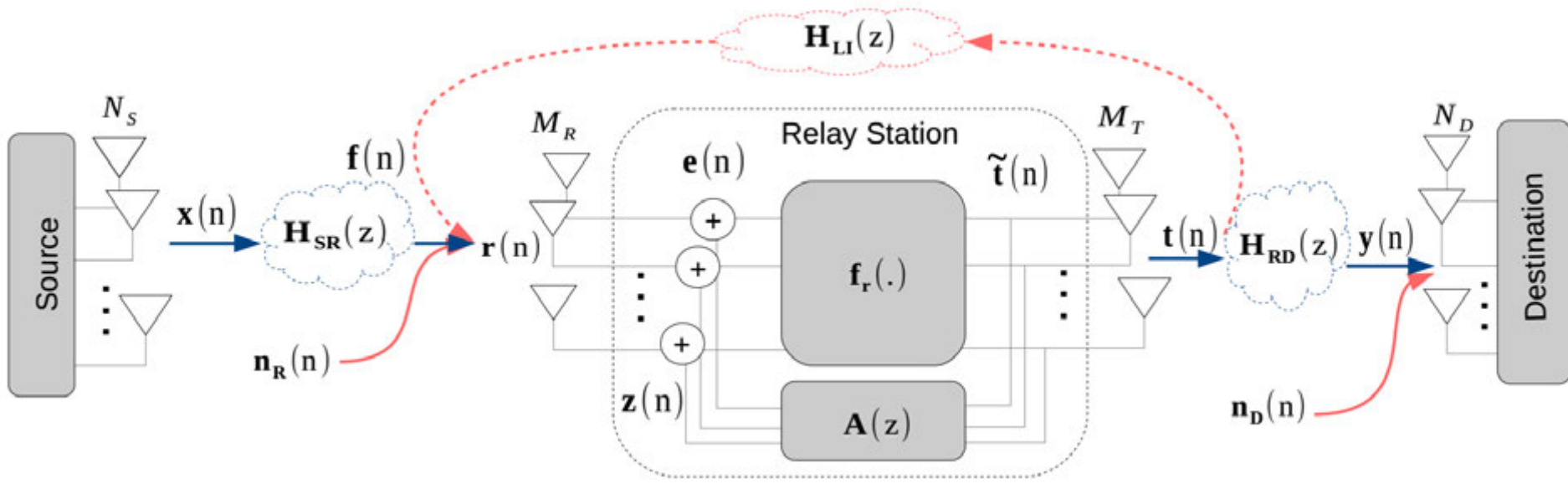
# In-band full-duplex relay station



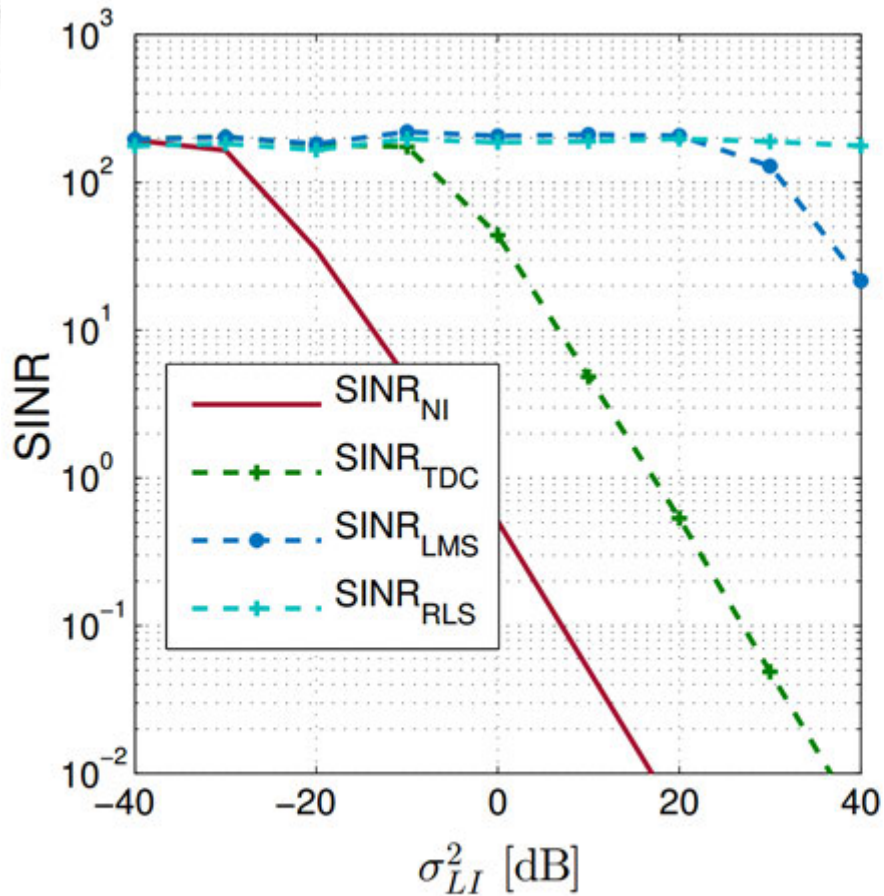
# Filter design: receive filters and transmit filters



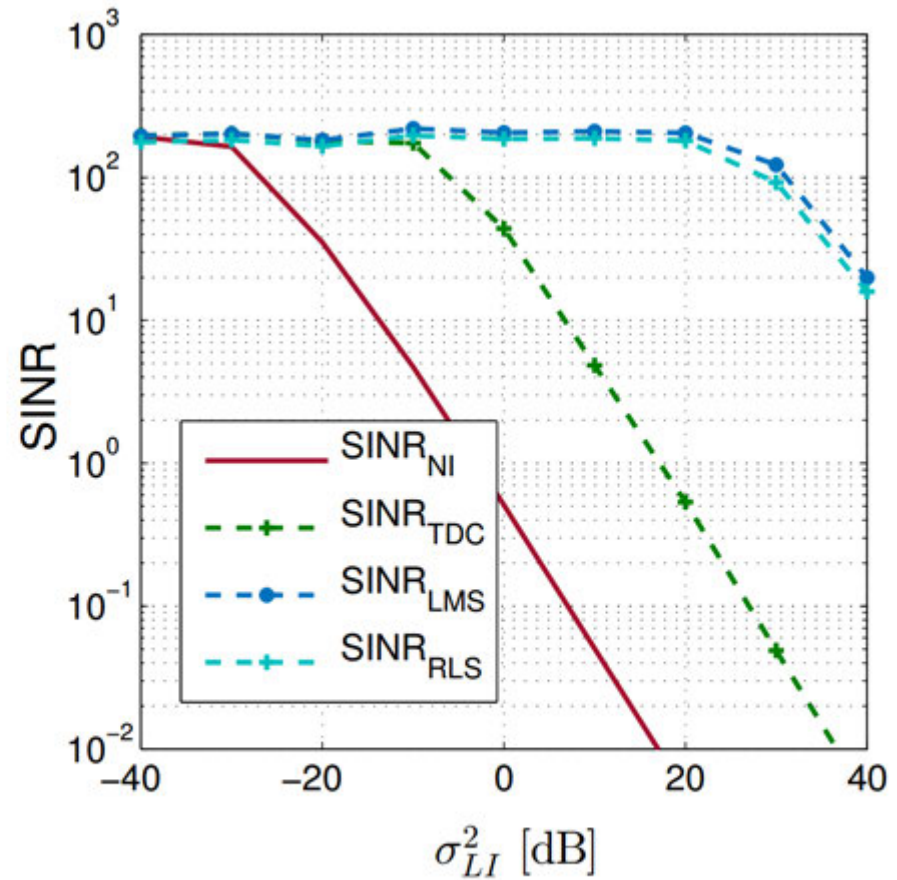
# Feedback filter for interference cancellation



# How loopback interference impacts on SINR

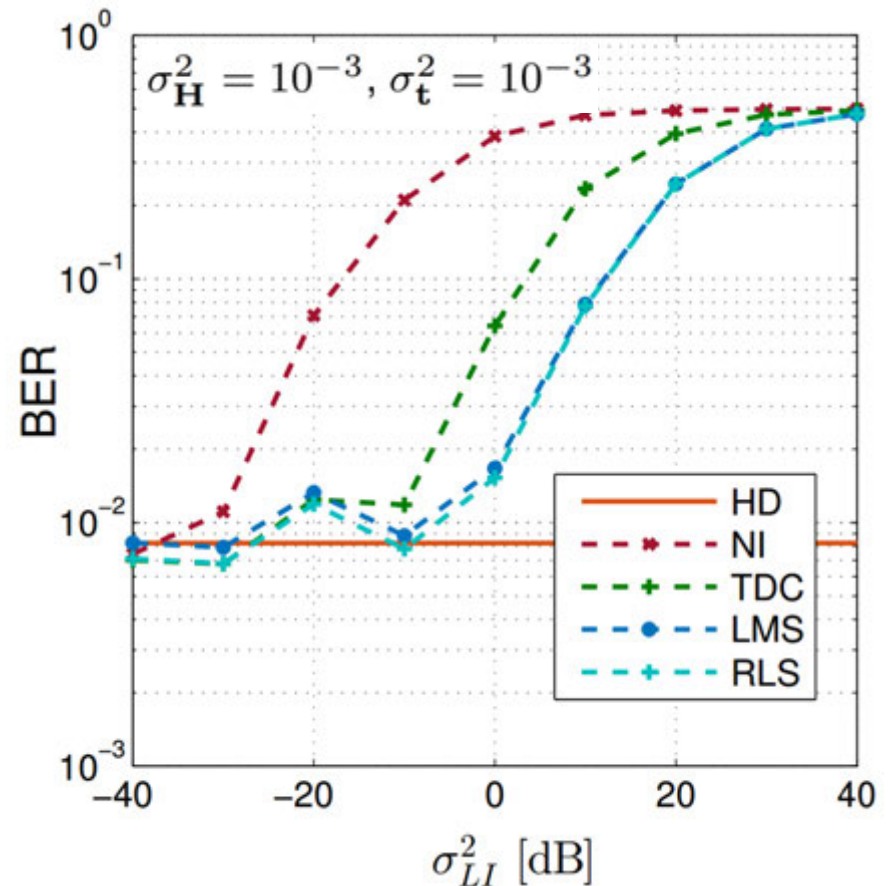
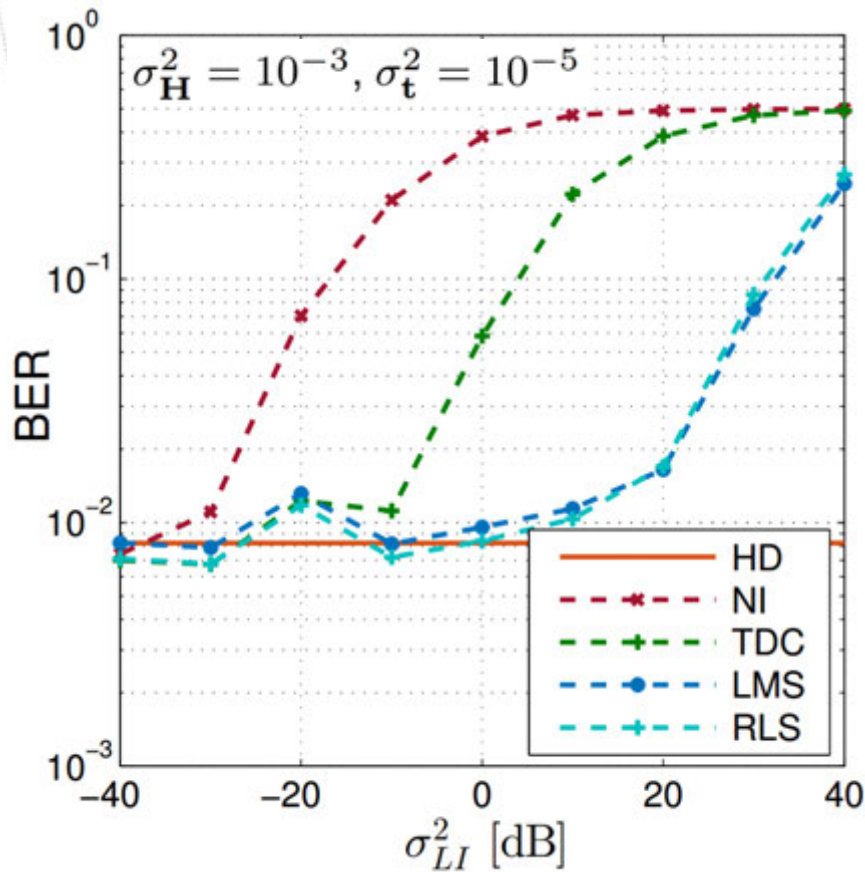


(a)  $\sigma_H^2 = 10^{-3}$ ,  $\sigma_t^2 = 10^{-5}$ .



(b)  $\sigma_H^2 = 10^{-3}$ ,  $\sigma_t^2 = 10^{-3}$ .

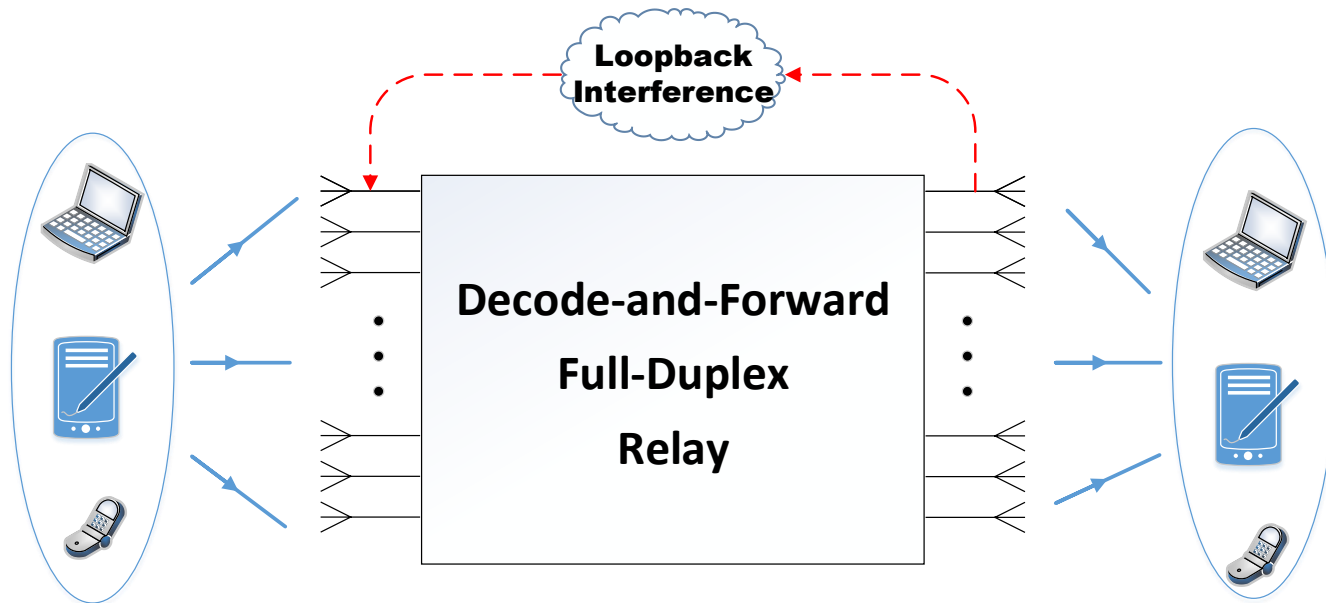
# Protection to self interference gain



Technique	Gain
Natural Isolation	Reference (0 dB)
NSP	-20 dB
MMSE	-25 dB
LMS	-30 to -40 dB
RLS	-30 to -40 dB
Perfect Cancellation	$-\infty$ dB

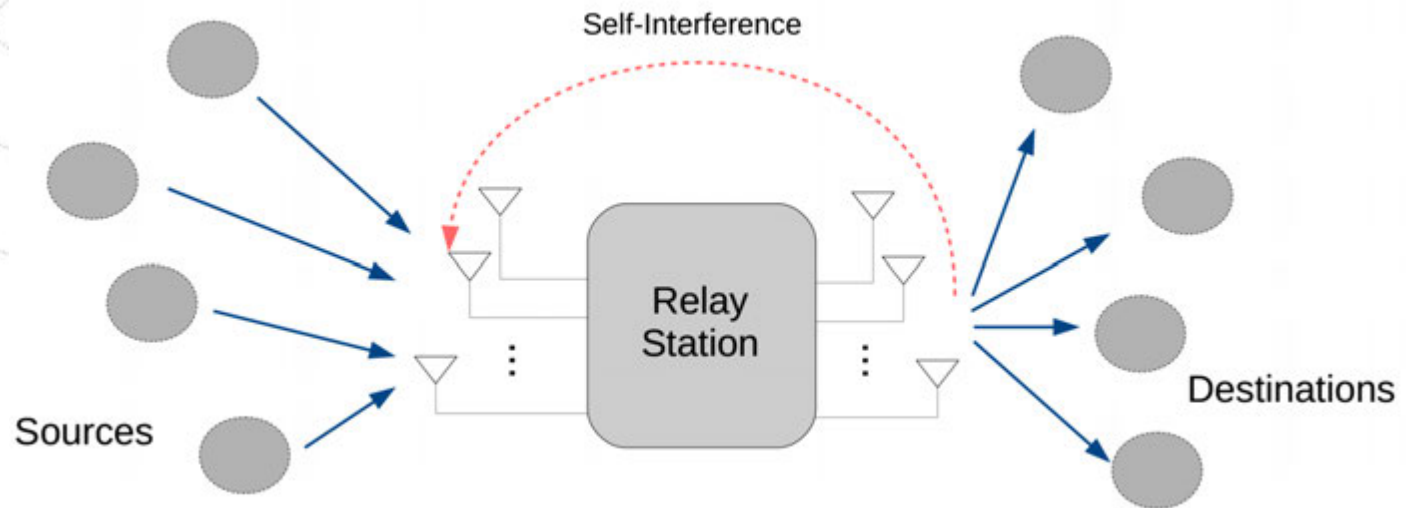


# Multi-pairs using virtual MIMO

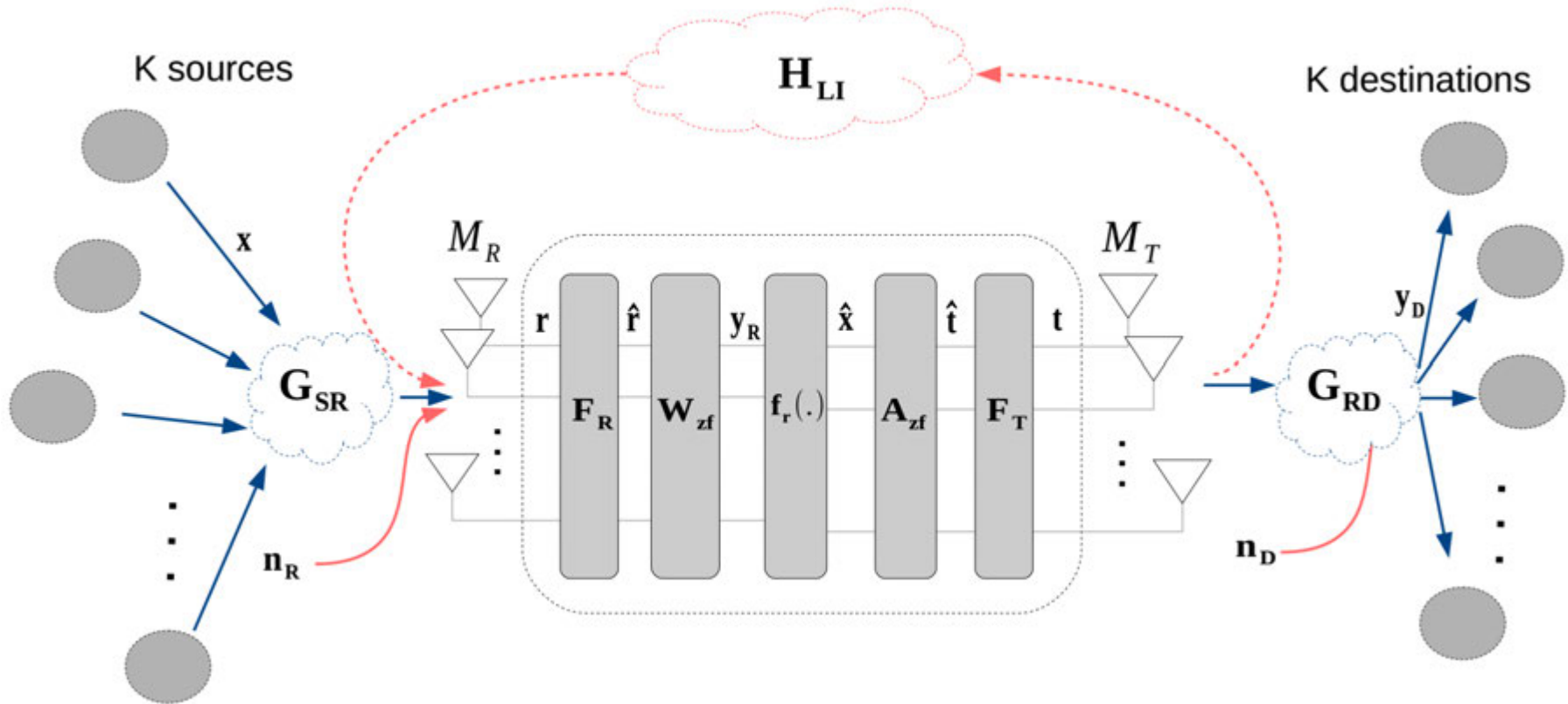




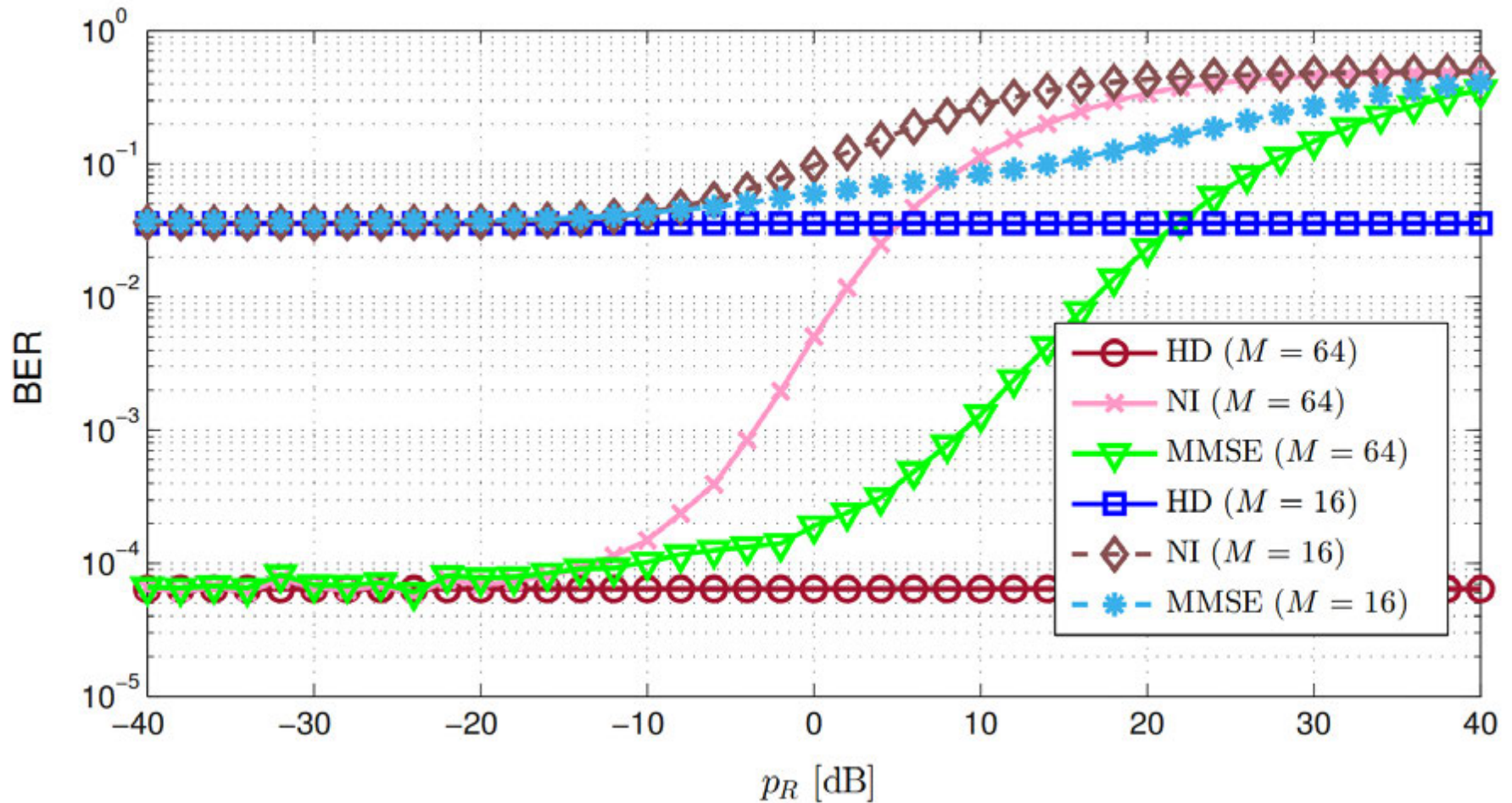
# Multi-pairs with a full-duplex relay (1)



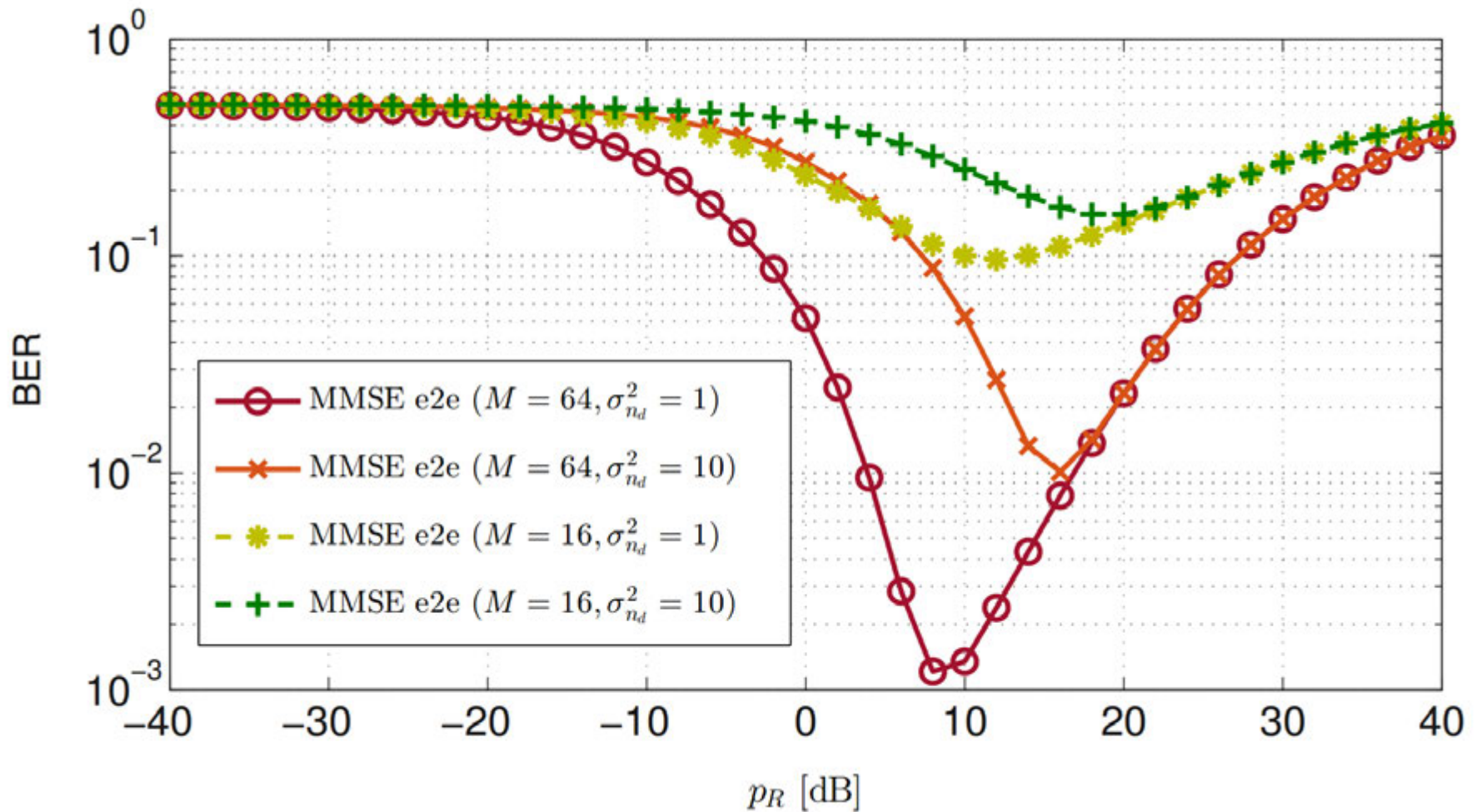
# Multi-pairs with a full-duplex relay (2)



# Multi-pairs with a full-duplex relay: loop interference gain

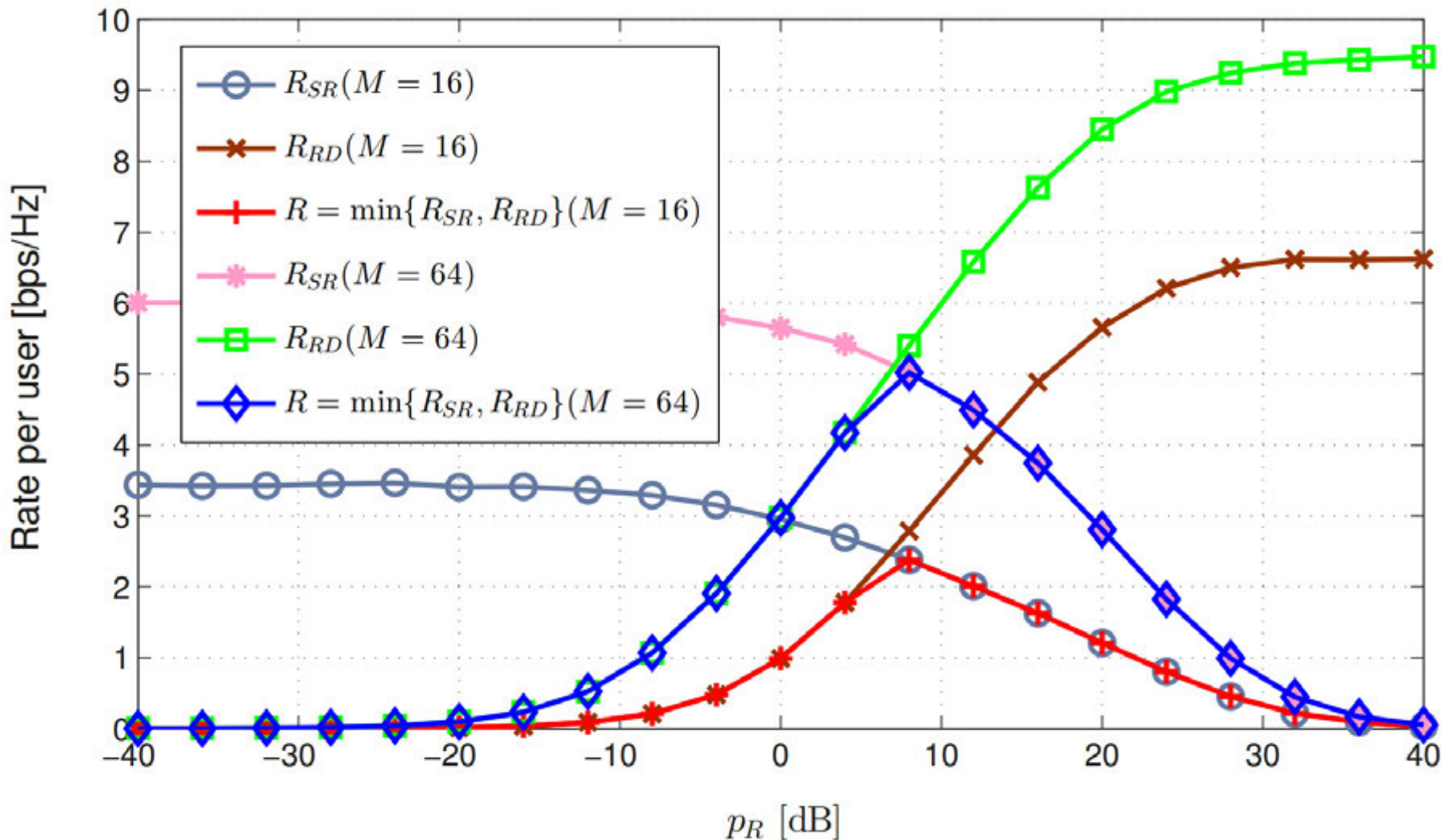


# Multi-pairs with a full-duplex relay: effect of the relay power on the performance





# Multi-pairs with a full-duplex relay: effect of the relay power on the rates



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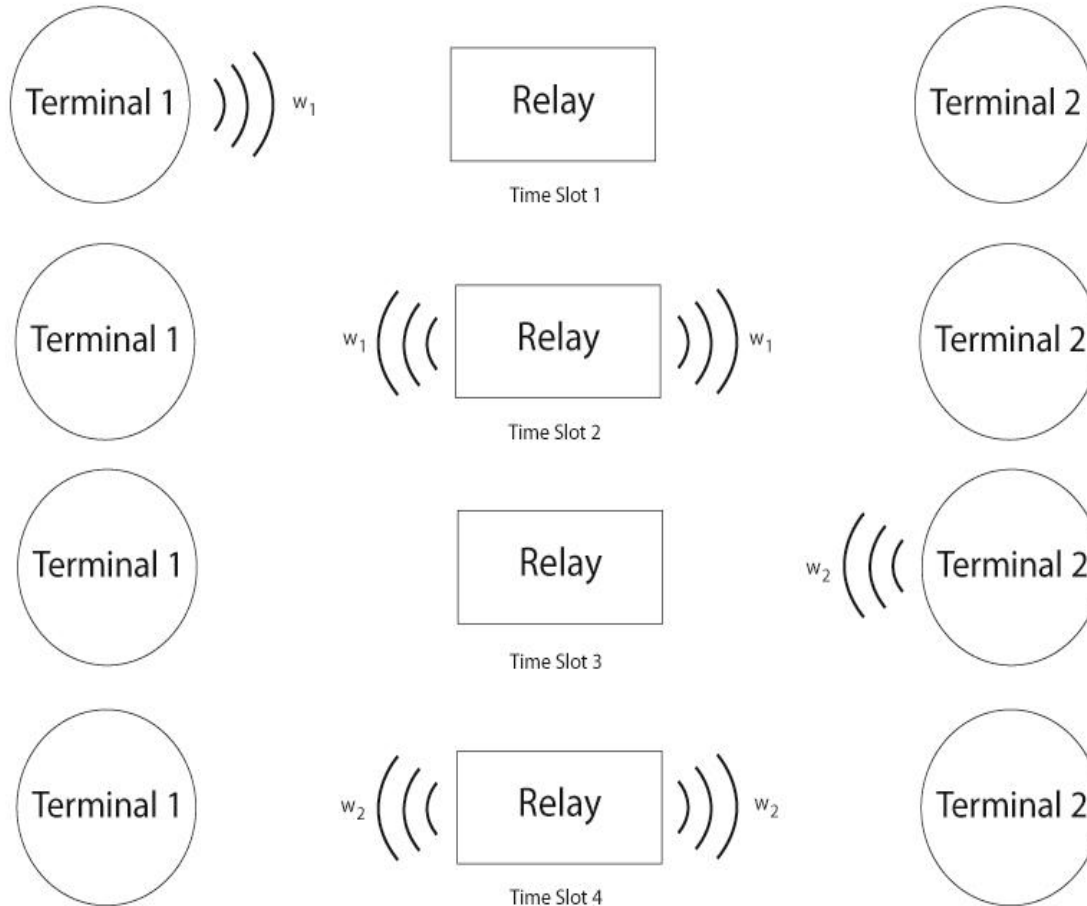
# Two way relay channel

Two way relay channel



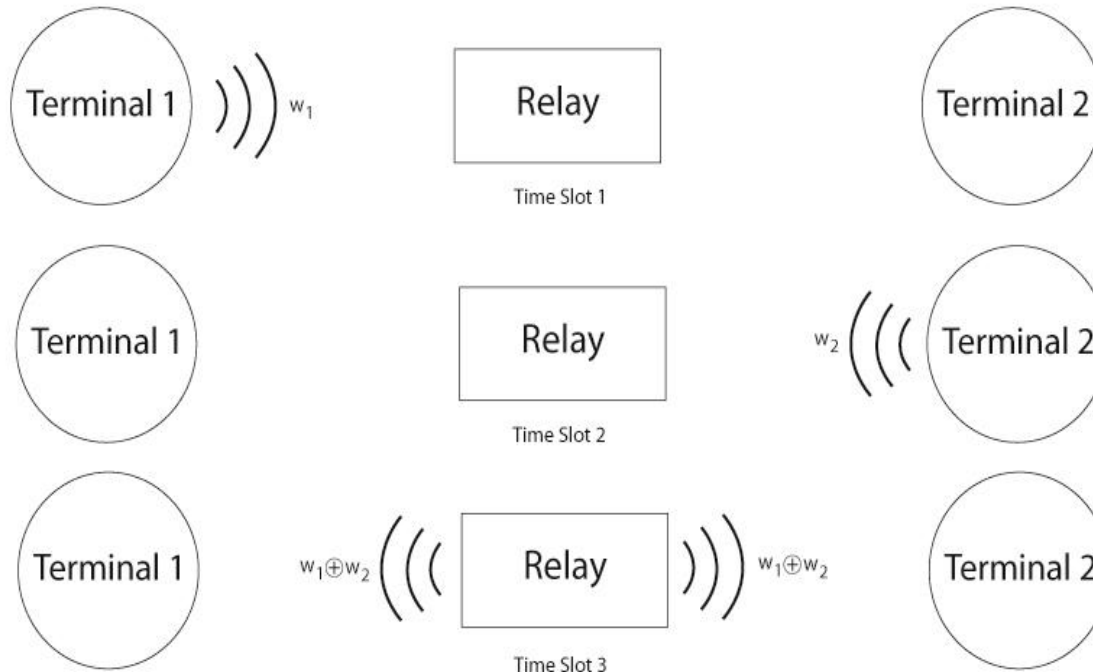
# Two way relay channel

## TDMA



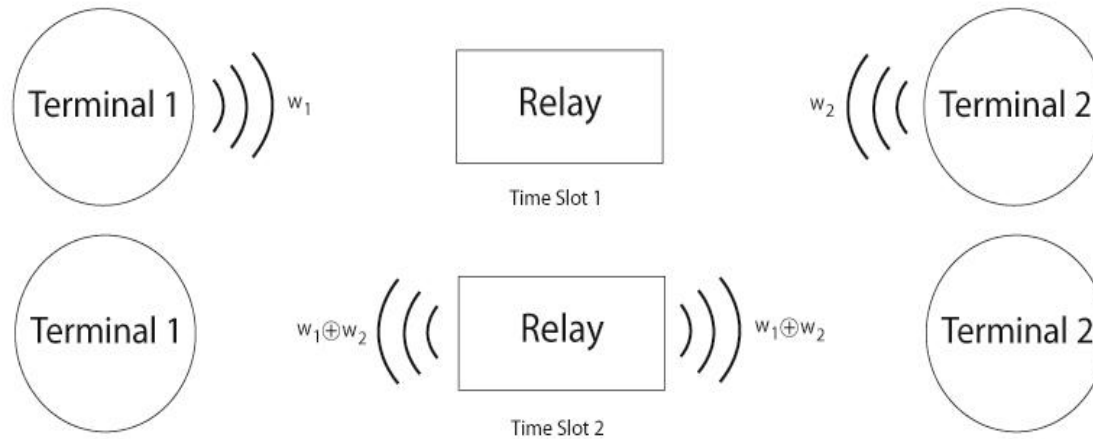
# Two way relay channel

## Network Coding



# Two way relay channel

## Physical layer Network Coding

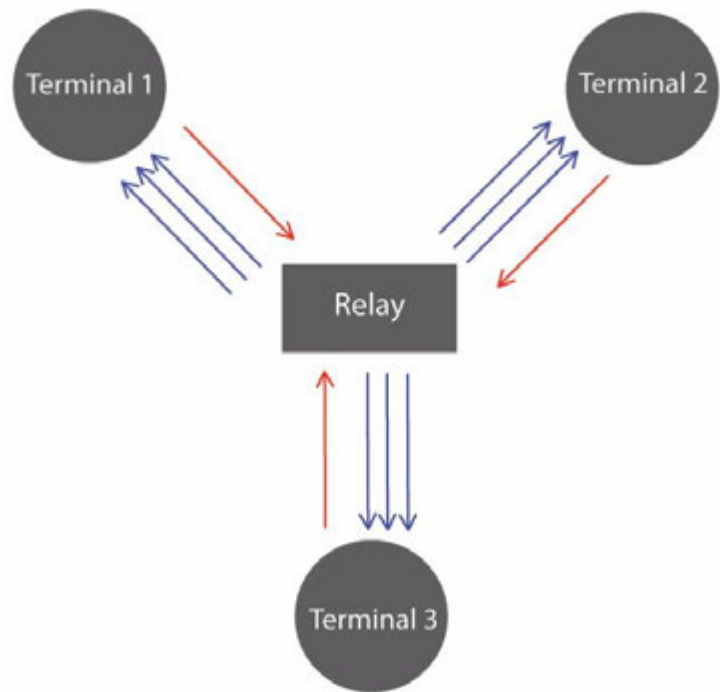
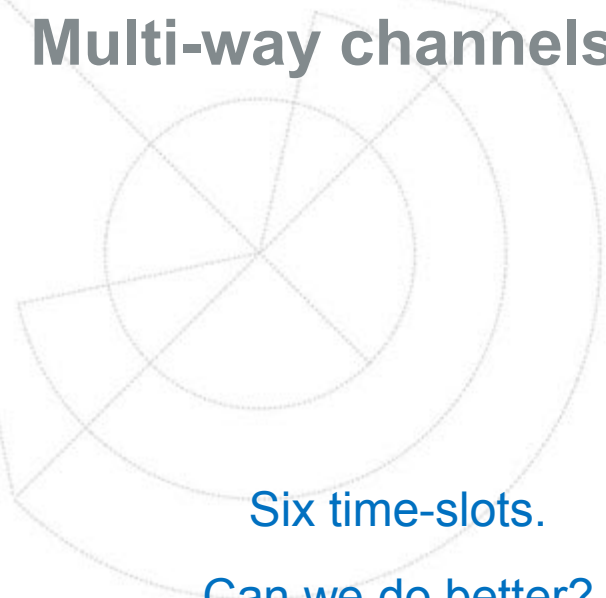


Two time-slots. Can we do better?

⇒ Merge both stages ⇔ in-band full-duplex.

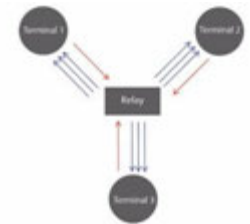


# Multi-way channels. The Y-network with TDMA:





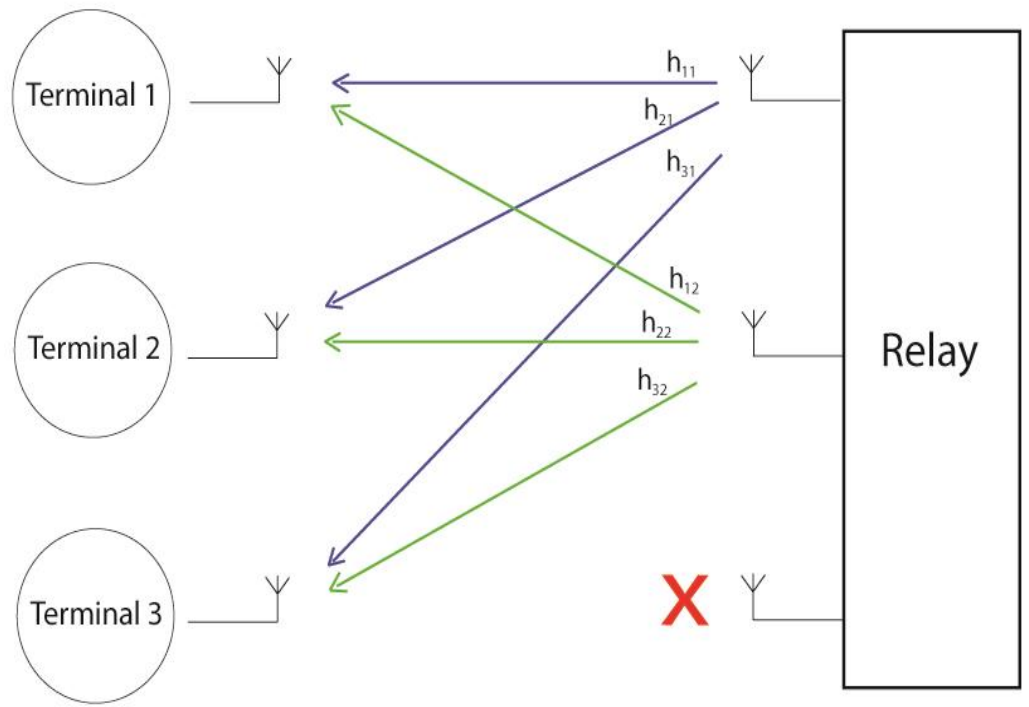
# Three time slots: a protocol with SISO (1)



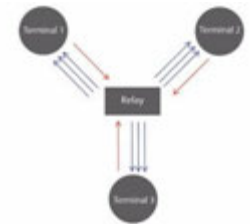
$$y_1[2] = h_{11}x_1 + h_{12}x_2 + n$$

$$y_2[2] = h_{21}x_1 + h_{22}x_2 + n$$

$$y_3[2] = h_{31}x_1 + h_{32}x_2 + n$$



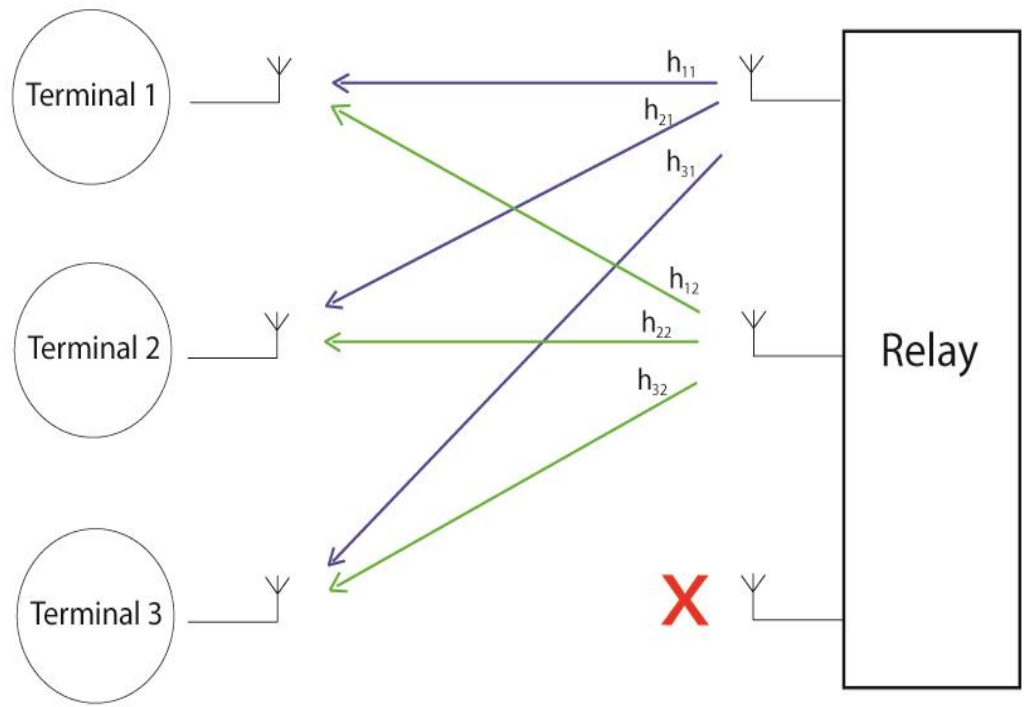
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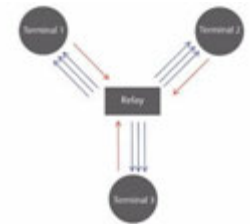
$$y_1[2] = \cancel{h_{11}x_1} + h_{12}x_2 + n$$

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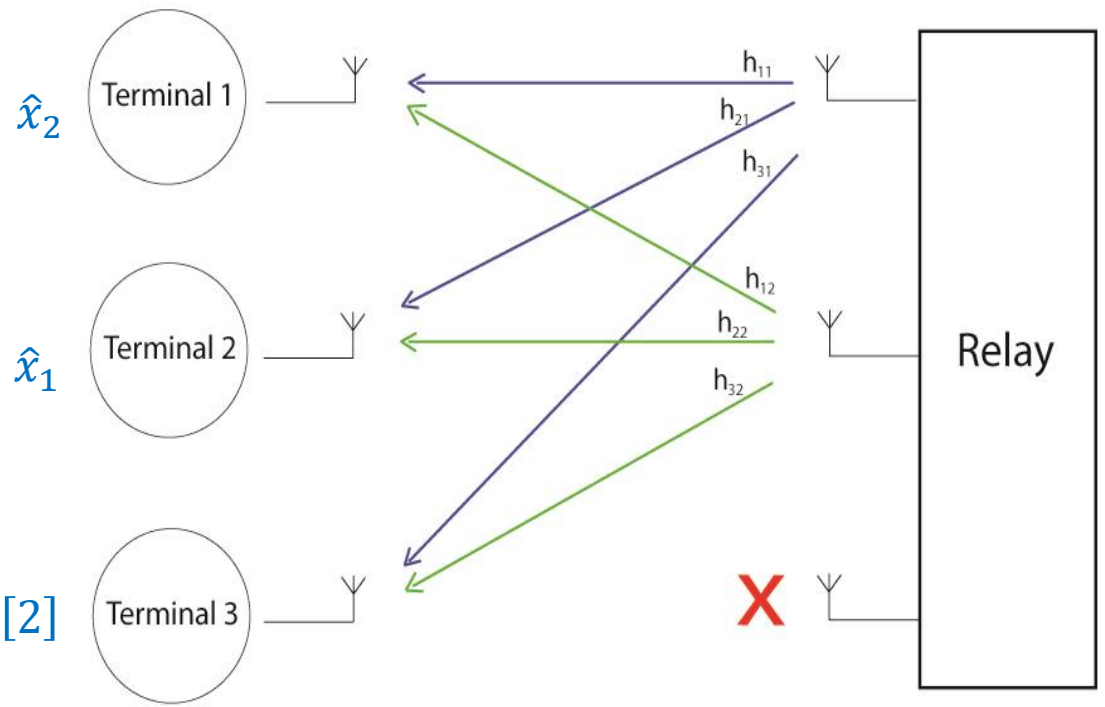
# Three time slots: a protocol with SISO (1)



$$y_1[2] = h_{11}x_1 + h_{12}x_2 + n$$

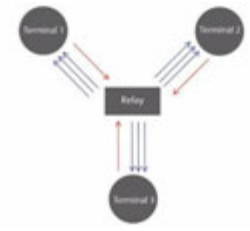
$$y_2[2] = h_{21}x_1 + h_{22}x_2 + n$$

$$y_3[2] = h_{31}x_1 + h_{32}x_2 + n \quad y_3[2]$$



X

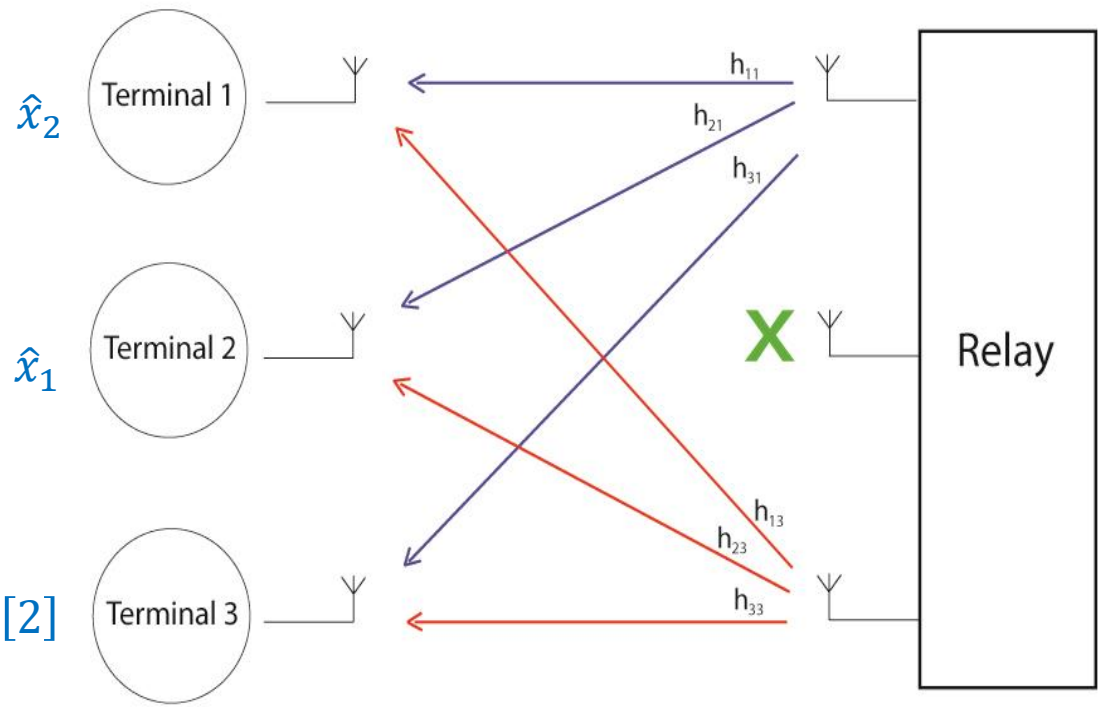
# Three time slots: a protocol with SISO (2)



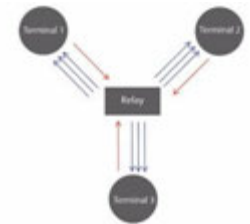
$$y_1[3] = h_{11}x_1 + h_{13}x_3 + n$$

$$y_2[3] = h_{21}x_1 + h_{23}x_3 + n$$

$$y_3[3] = h_{31}x_1 + h_{33}x_3 + n \quad y_3[2]$$



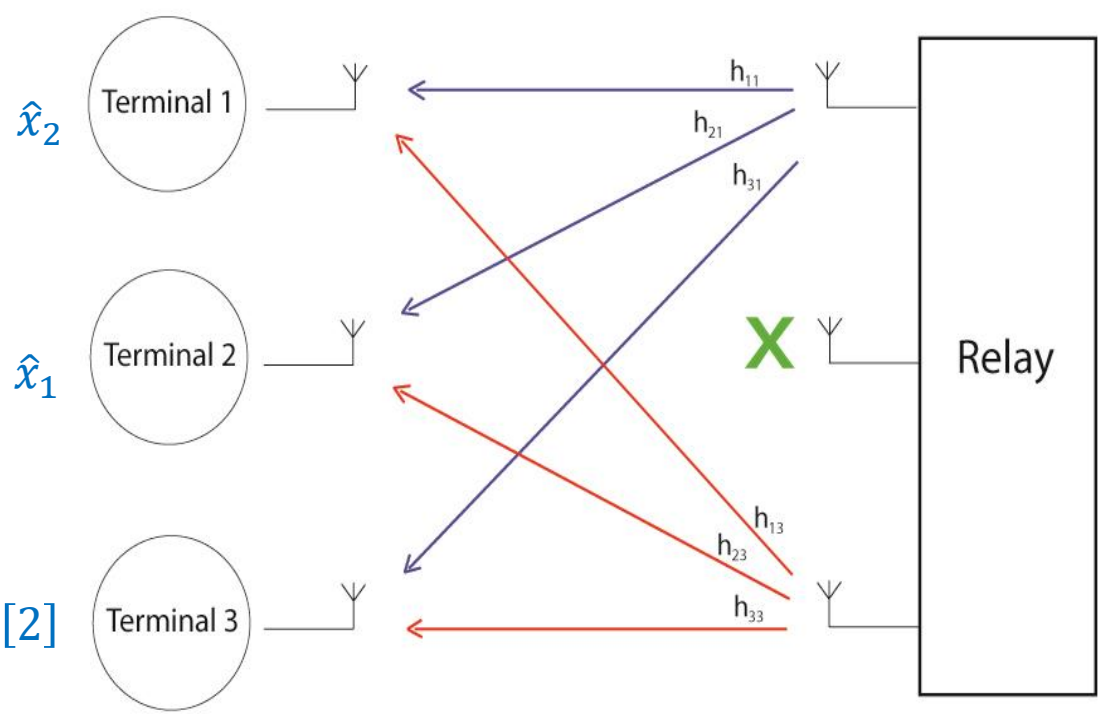
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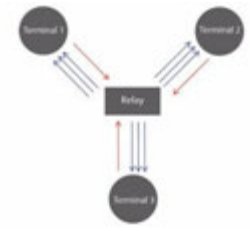
$$y_1[3] = \cancel{h_{11}x_1} + h_{13}x_3 + n$$

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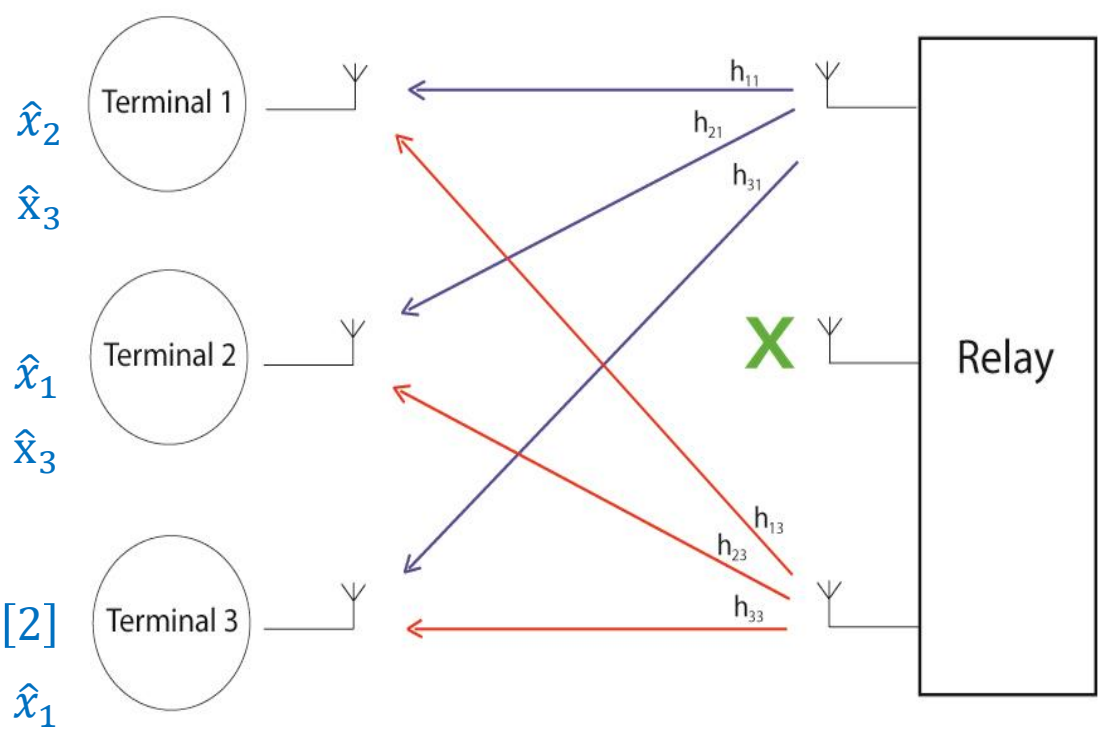
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$\hat{x}_2$

$\hat{x}_3$

$\hat{x}_1$

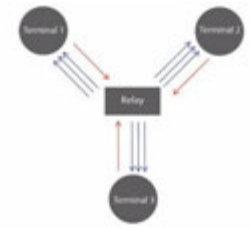
$\hat{x}_3$

$y_3[2]$

$\hat{x}_1$



# Three time slots: a protocol with SISO (2)

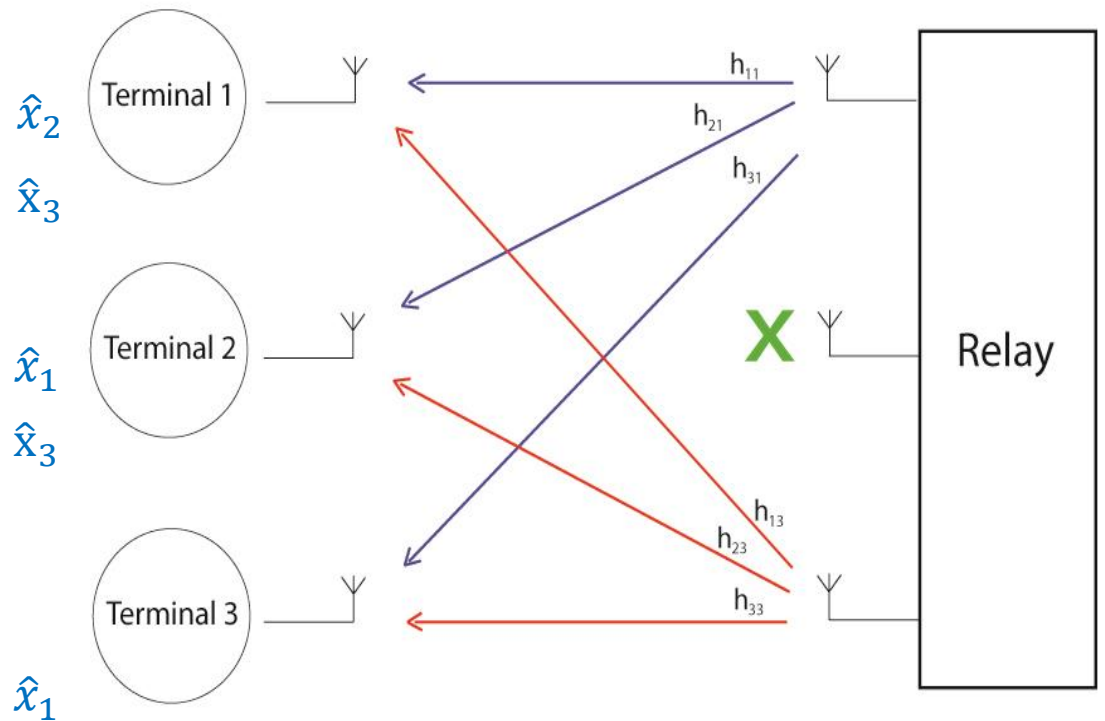


$$y_1[3] = \cancel{h_{11}}x_1 + h_{13}x_3 + n$$

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$\hat{x}_2$

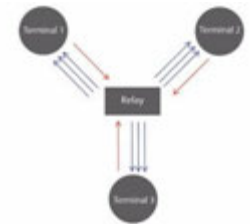
$\hat{x}_3$

$\hat{x}_1$

$\hat{x}_3$

$\hat{x}_1$

# Three time slots: a protocol with SISO (2)

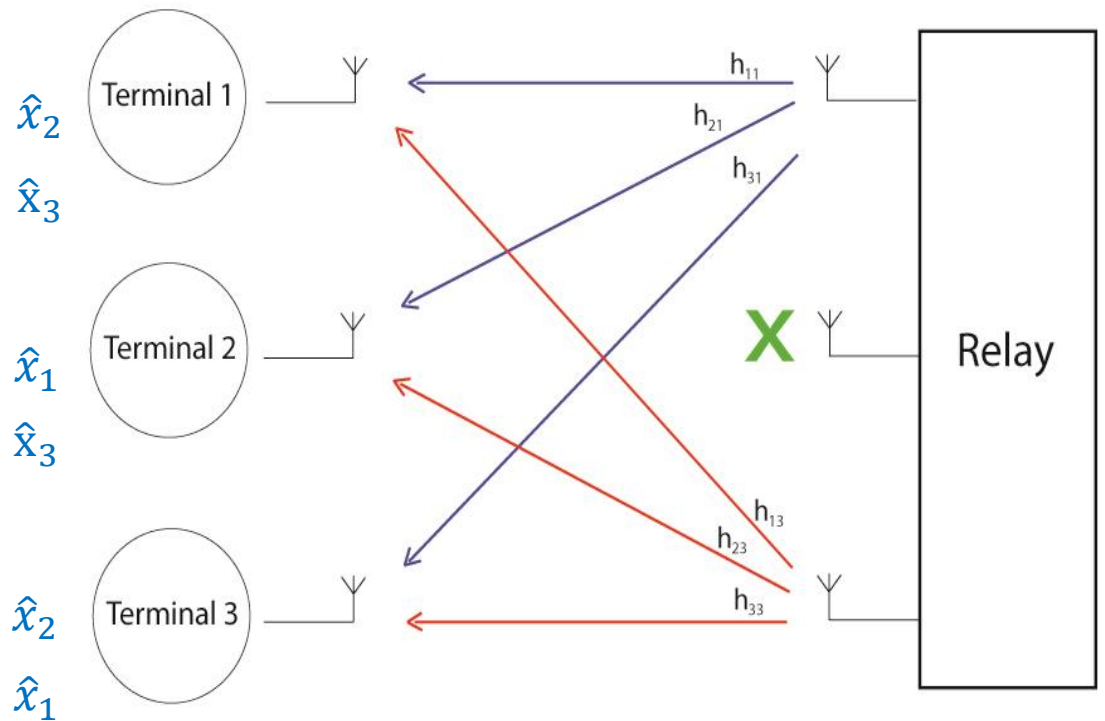


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$$y_3[2] = \cancel{h_{21}}x_1 + h_{32}x_2 + n$$



$\hat{x}_2$

$\hat{x}_3$

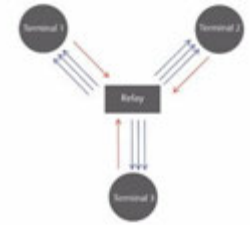
$\hat{x}_1$

$\hat{x}_3$

$\hat{x}_2$

$\hat{x}_1$

# Two time slots: a protocol with SISO terminals

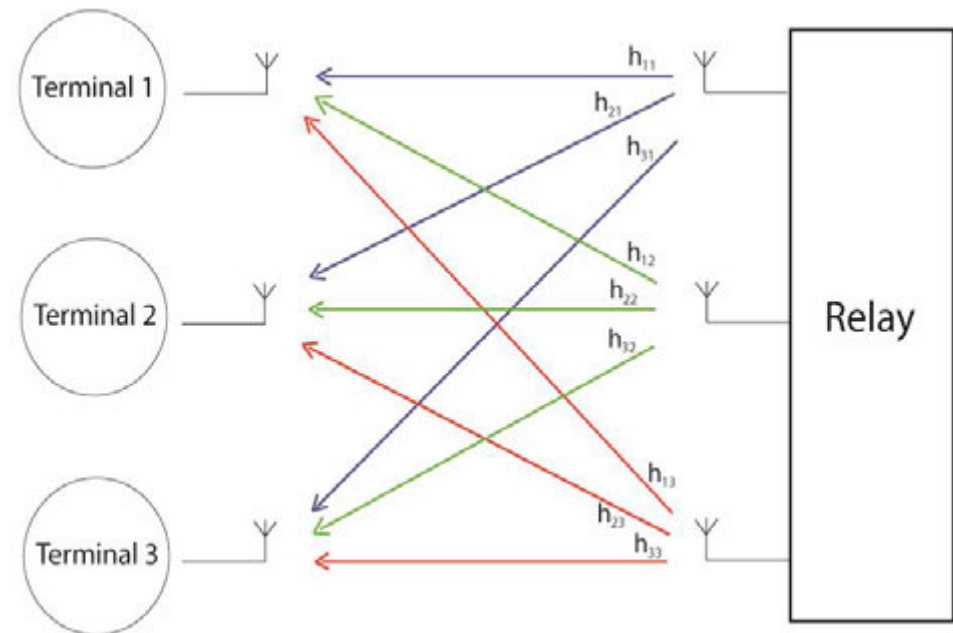


- Terminals have CSIR

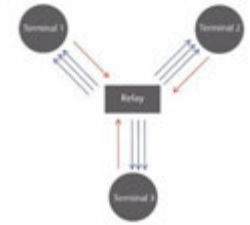
$$y_1 = h_{11}x_1 + h_{12}x_2 + h_{13}x_3 + n$$

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$$y_3 = h_{31}x_1 + h_{32}x_2 + h_{33}x_3 + n$$



# Two time slots: a protocol with SISO terminals

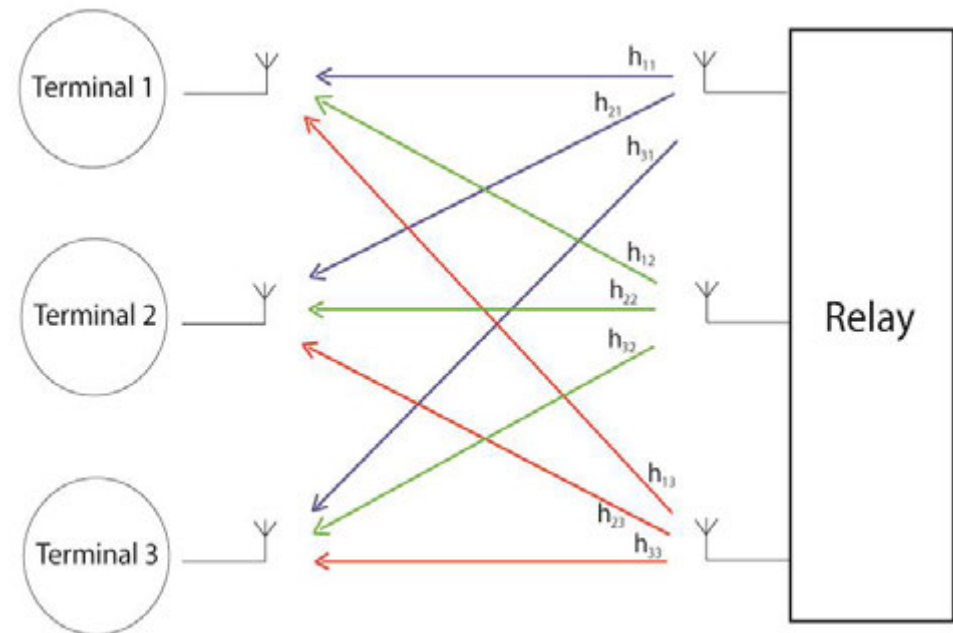


- Each terminal cancels its own message

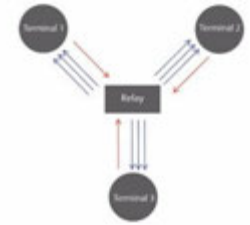
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# Two time slots: a protocol with SISO terminals

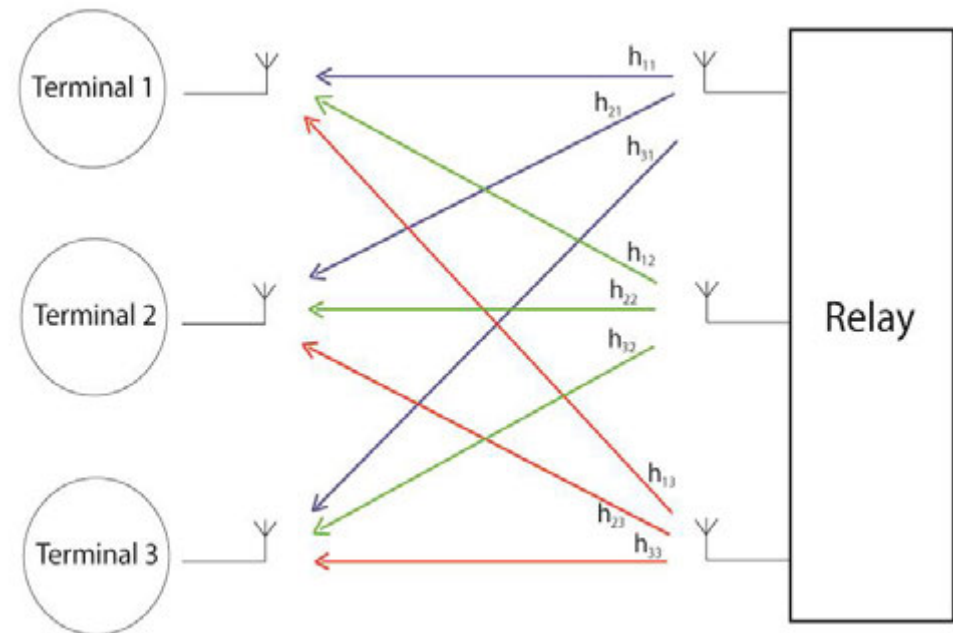


- Joint detection (ML) to detect the last two messages

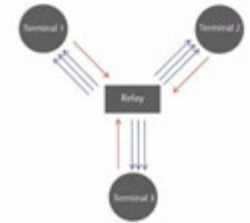
$$y'_1 = y_1 - h_{11}x_1$$

$$y'_2 = y_2 - h_{22}x_2$$

$$y'_3 = y_3 - h_{33}x_3$$



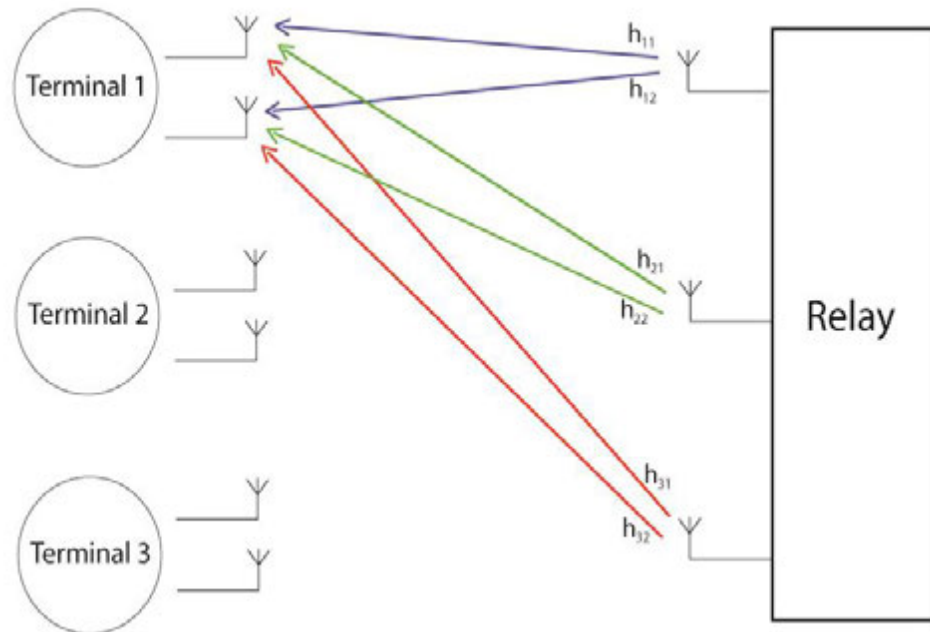
# Two time slots: a protocol with MIMO terminals



Each terminal cancels its own message and the remaining two are MIMO detected.

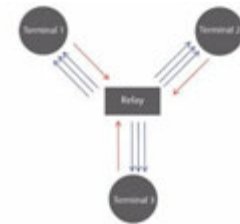
$$y_{11} = h_{11}x_1 + h_{21}x_2 + h_{31}x_3 + n$$

$$y_{12} = h_{12}x_1 + h_{22}x_2 + h_{32}x_3 + n$$



Two time-slots only: Can we do better?



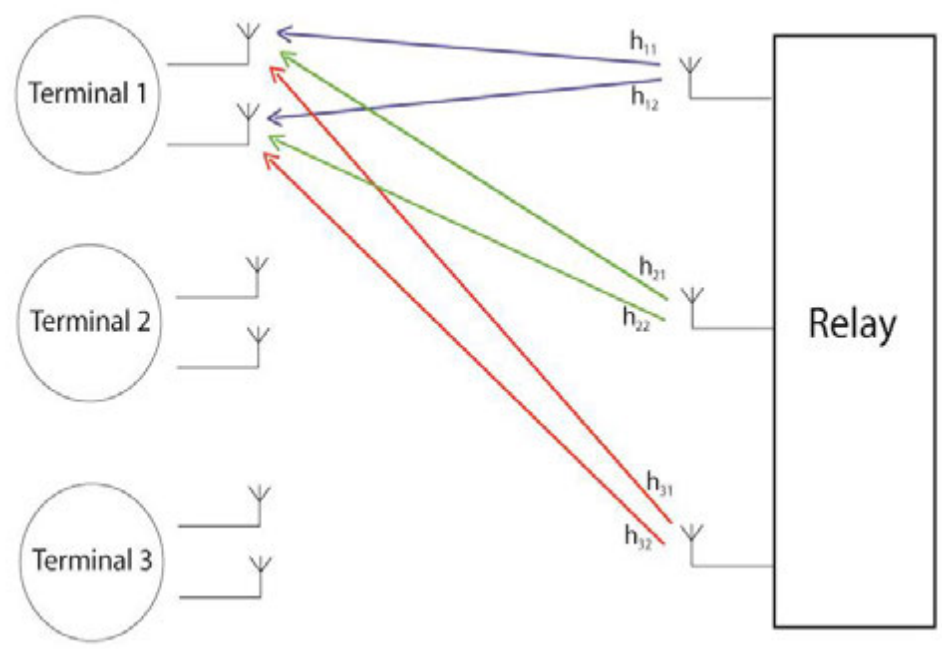


# Two time slots: a protocol with MIMO terminals

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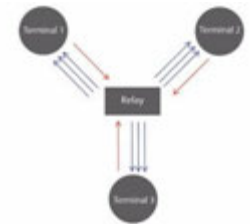
$$y_{11} = \cancel{h_{11}}x_1 + h_{21}x_2 + h_{31}x_3 + n$$

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Two time-slots only: Can we do better?

# Two time slots: a protocol with MIMO terminals



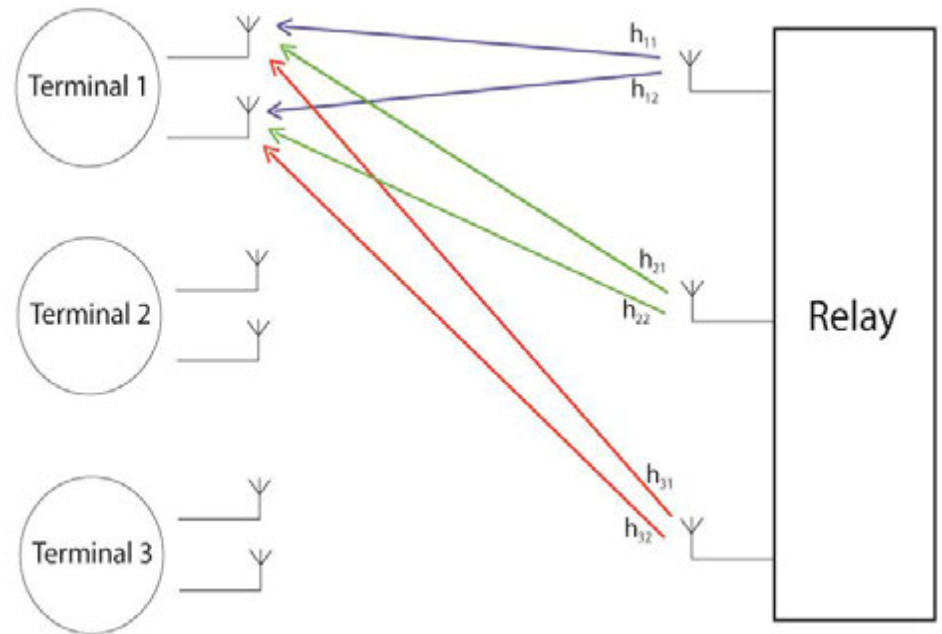
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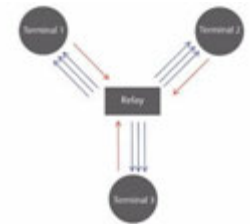
- A 2x2 MIMO detection problem:

$$\begin{bmatrix} y'_{11} \\ y'_{12} \end{bmatrix} = \begin{bmatrix} h_{21} & h_{31} \\ h_{22} & h_{32} \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} + [\mathbf{n}]$$

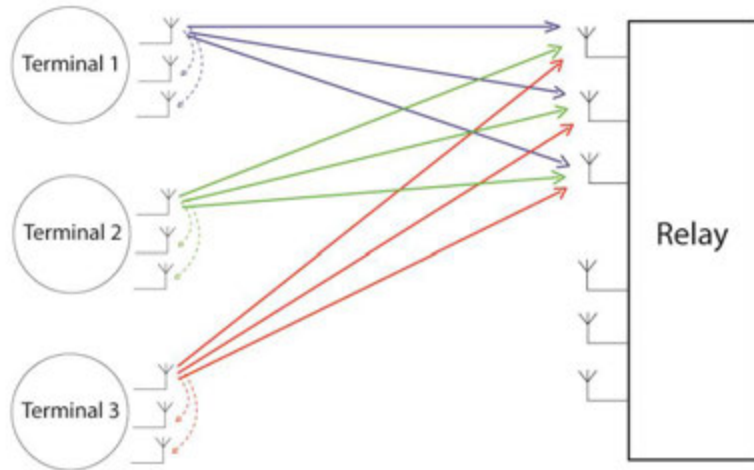


Two time-slots only: Can we do better?

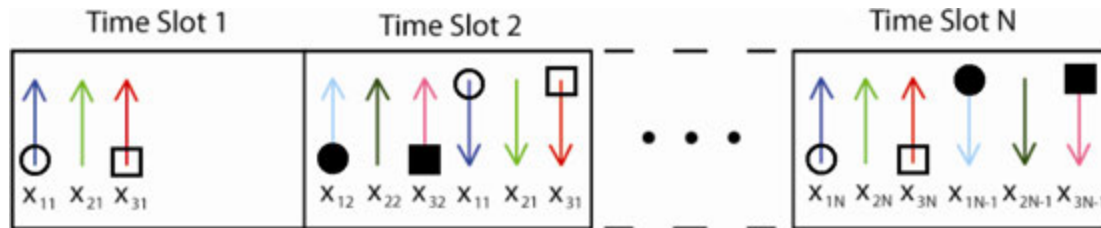
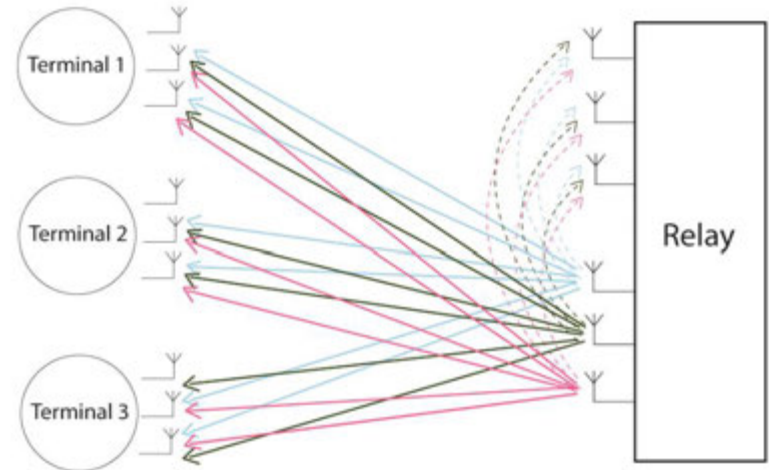
# Full-duplex in the Y-channel: how it can work



Uplink

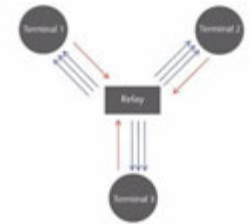


Downlink

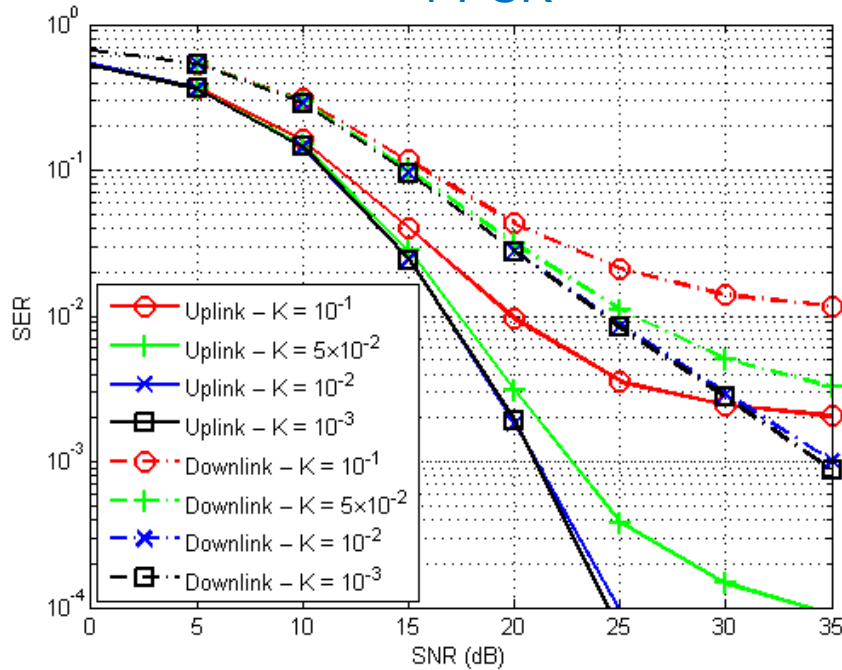


One time slot only on average as the number of messages exchanges increases!

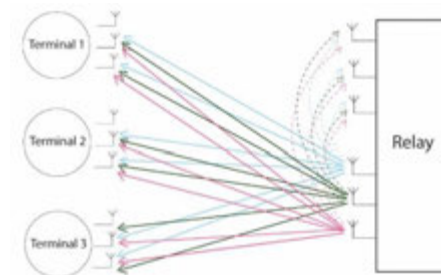
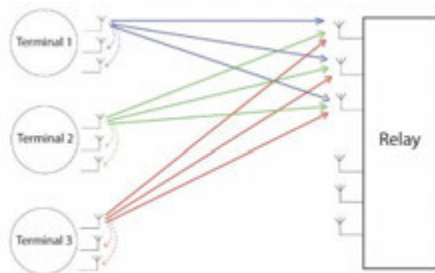
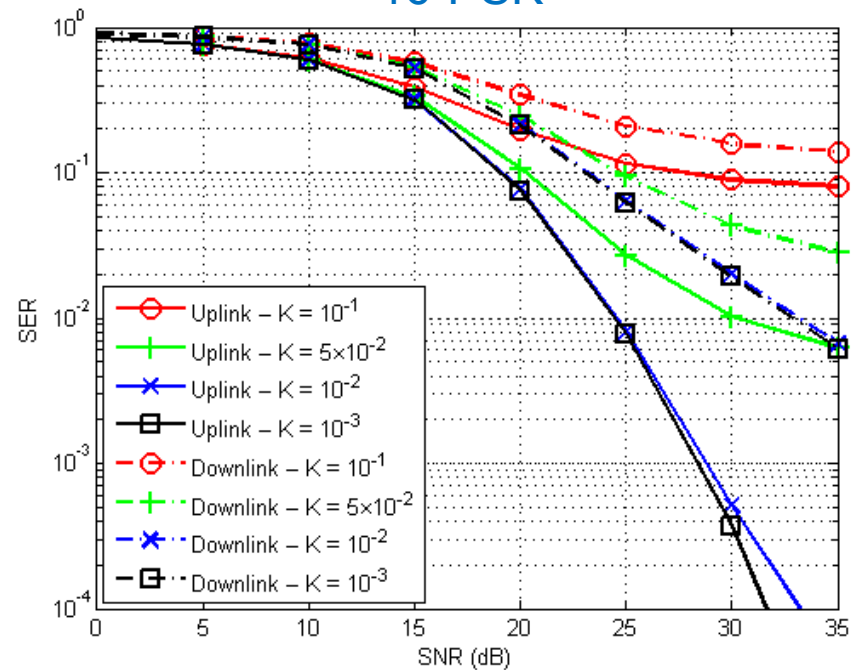
# Full-duplex in the Y-channel: Performance



4-PSK



16-PSK



# *Signal processing in the upcoming wireless networks:*

*Untangling signals in space, in spectrum, and network coded*

**1- Spatial multiplexing (MIMO)**

**2- Massive MIMO**

**3- In-band full-duplex**

**4- TWRC and the Y-network**

**5- Full-duplex + massive MIMO + PLNC**

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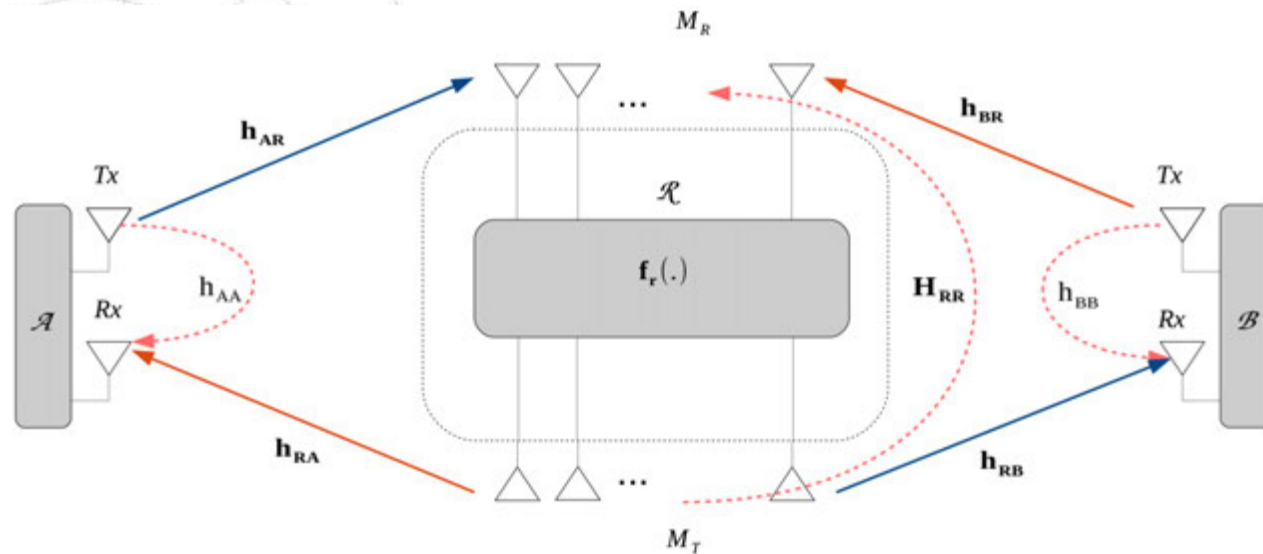
**3- In-band full-duplex**

**4- TWRC and the Y-network**

**5- Full-duplex + massive MIMO + PLNC**



# Full-duplex with a massive MIMO relay and PLNC

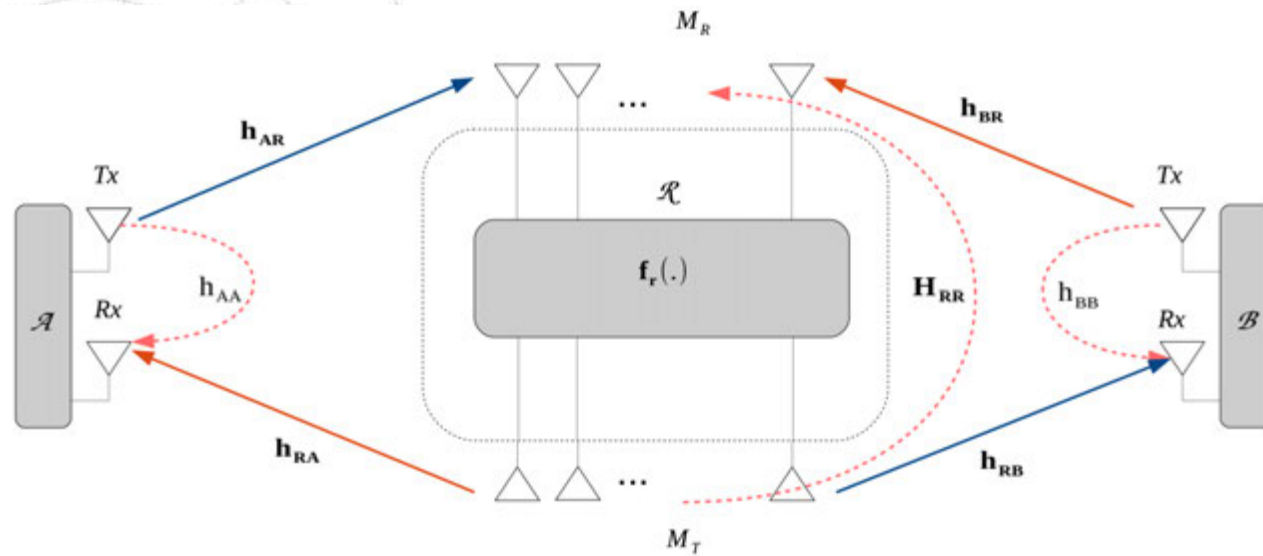


$$\mathbf{y}_R(n) = \sqrt{p_A} \mathbf{h}_{AR} x_A(n) + \sqrt{p_B} \mathbf{h}_{BR} x_B(n) + \sqrt{p_R} k_R \mathbf{H}_{RR} \mathbf{x}_R(n) + \mathbf{n}_R(n)$$

$$y_A(n) = \sqrt{p_R} \mathbf{h}_{RA} \mathbf{x}_R(n) + \sqrt{p_A} k_A h_{AA} x_A(n) + n_A(n)$$

$$y_B(n) = \sqrt{p_R} \mathbf{h}_{RB} \mathbf{x}_R(n) + \sqrt{p_B} k_B h_{BB} x_B(n) + n_B(n)$$

# Self-interference model



$$\begin{aligned} & \mathbf{H}_{RR}\mathbf{x}_R(n) - \widehat{\mathbf{H}}_{RR}\widehat{\mathbf{x}}_R(n) \\ &= \left( \tilde{\mathbf{H}}_{RR} + \mathcal{E}_{\mathbf{H}_{RR}} \right) \left( \tilde{\mathbf{x}}_R(n) + \mathcal{E}_{\mathbf{x}_R}(n) \right) - \widehat{\mathbf{H}}_{RR}\widehat{\mathbf{x}}_R(n) \\ &\triangleq k_R \mathbf{H}_{RR}\mathbf{x}_R(n) \end{aligned}$$

$$h_{AA}x_A(n) - \widehat{h_{AA}x_A(n)} \triangleq k_A h_{AA}x_A(n)$$

$$h_{BB}x_B(n) - \widehat{h_{BB}x_B(n)} \triangleq k_B h_{BB}x_B(n)$$

	$(k_A, k_B, k_R)$
Natural isolation	Reference (0 dB)
Conventional time-domain cancellation	-20 dB
Recursive least squares cancellation	-30 to -40 dB
Perfect cancellation	$-\infty$ dB

## Denoise and forward with QPSK in the TWRC

$$\mathcal{M} : \mathbb{Z}_Q \rightarrow \mathcal{D}_Q \quad S \in \mathbb{Z}_Q = \{0, 1, \dots, Q - 1\}$$

QPSK modulation uses:  $\mathbb{Z}_4 = \{0, 1, 2, 3\}$

Decision at the relay:

$$(\hat{S}_A, \hat{S}_B) = \underset{(s_1, s_2) \in \mathbb{Z}_Q^2}{\operatorname{argmin}} \quad \|\mathbf{y}_R(n) - (\mathbf{h}_{AR}\mathcal{M}(s_1) + \mathbf{h}_{BR}\mathcal{M}(s_2))\|^2$$

Decisions at the terminals:

$$\hat{S}'_B = \underset{s \in \mathbb{Z}_Q}{\operatorname{argmin}} \quad \|\mathbf{y}_A(n) - \mathbf{h}_{RA}\mathcal{M}_R(\mathcal{C}(S_A, s))\|^2$$

$$\hat{S}'_A = \underset{s \in \mathbb{Z}_Q}{\operatorname{argmin}} \quad \|\mathbf{y}_B(n) - \mathbf{h}_{RB}\mathcal{M}_R(\mathcal{C}(S_B, s))\|^2$$

## Two mappings: GF(2) and GF(4)

$\mathcal{C}(s_1, s_2) \neq \mathcal{C}(s'_1, s_2)$  for any  $s_1 \neq s'_1 \in \mathbb{Z}_Q$  and  $s_2 \in \mathbb{Z}_Q$

$\mathcal{C}(s_1, s_2) \neq \mathcal{C}(s_1, s'_2)$  for any  $s_1 \in \mathbb{Z}_Q$  and  $s_2 \neq s'_2 \in \mathbb{Z}_Q$

$$\mathcal{C} : \mathbb{Z}_Q^2 \rightarrow \mathbb{Z}_Q, \mathcal{C}(S_1, S_2) = S_1 \oplus S_2$$

$$\mathcal{C} : \mathbb{Z}_Q^2 \rightarrow \mathbb{Z}_Q, \mathcal{C}(S_1, S_2) = [S_1 + S_2] \bmod Q$$

# Two mappings: GF(2) and GF(4)

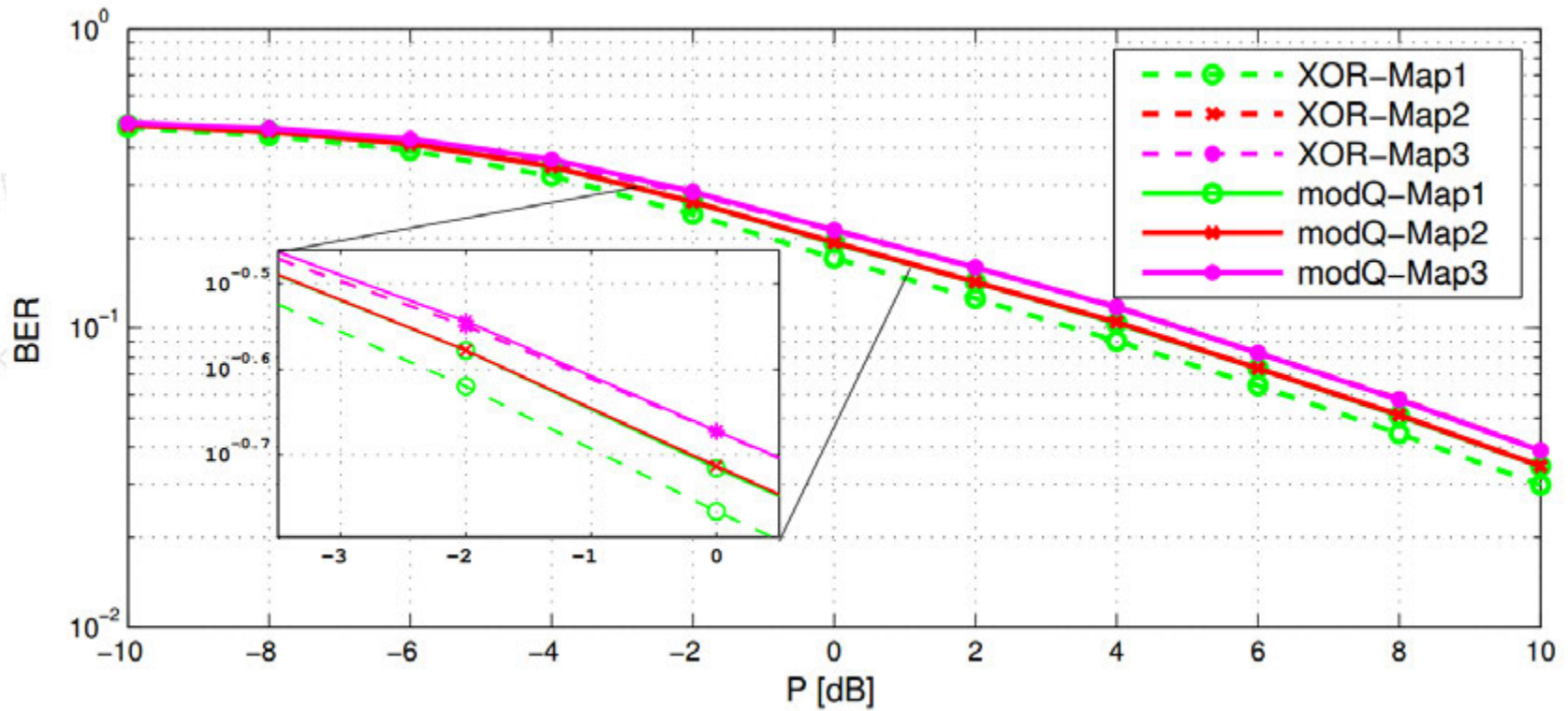
	$C(S_1, S_2) = S_1 \oplus S_2$				$C(S_1, S_2) = [S_1 + S_2] \bmod Q$				
Pair message $(S_A, S_B)$	(0,0)	(0,1)	(0,2)	(0,3)	(0,0)	(0,1)	(0,2)	(0,3)	
	(1,1)	(1,0)	(1,3)	(1,2)	(1,3)	(1,0)	(1,1)	(1,2)	
	(2,2)	(2,3)	(2,0)	(2,1)	(2,2)	(2,3)	(2,0)	(2,1)	
	(3,3)	(3,2)	(3,1)	(3,0)	(3,1)	(3,2)	(3,3)	(3,0)	
Code $S_R = C(S_A, S_B)$	0	1	2	3	0	1	2	3	
Mapping $\mathcal{M}_{R,4}$ :	1)	$+1 + j$	$+1 - j$	$-1 + j$	$-1 - j$	$+1 + j$	$+1 - j$	$-1 + j$	$-1 - j$
	2)	$+1 + j$	$-1 - j$	$+1 - j$	$-1 + j$	$+1 + j$	$-1 - j$	$+1 - j$	$-1 + j$
	3)	$+1 + j$	$+1 - j$	$-1 - j$	$-1 + j$	$+1 + j$	$+1 - j$	$-1 - j$	$-1 + j$

# Bit error probability

$$\begin{aligned}
 P_{BER,A} &= P(\hat{S}'_2 \neq S_2) = \sum_{S_1 \in \mathbb{Z}_Q} P(S_1) \left[ \frac{1}{Q \log_2 Q} \sum_{X \in \mathcal{D}_Q} \sum_{\tilde{X} \neq X \in \mathcal{D}_Q} P(X \rightarrow \tilde{X}) \right. \\
 &\quad \left. \times d_H(B_Q(x : \mathcal{M}_{R,Q}(\mathcal{C}(S_1, x)) = X), B_Q(x : \mathcal{M}_{R,Q}(\mathcal{C}(S_1, x)) = \tilde{X})) \right] \\
 &= \frac{1}{Q \log_2 Q} \sum_{X \in \mathcal{D}_Q} \sum_{\tilde{X} \neq X \in \mathcal{D}_Q} P(X \rightarrow \tilde{X}) \times \\
 &\quad \left[ \sum_{S_1 \in \mathbb{Z}_Q} P(S_1) \times d_H(B_Q(x : \mathcal{M}_{R,Q}(\mathcal{C}(S_1, x)) = X), B_Q(x : \mathcal{M}_{R,Q}(\mathcal{C}(S_1, x)) = \tilde{X})) \right] \\
 &\quad \underbrace{\hspace{10em}}_{\triangleq N_B(X \rightarrow \tilde{X})} \\
 &= \frac{1}{Q \log_2 Q} \sum_{X \in \mathcal{D}_Q} \sum_{\tilde{X} \neq X \in \mathcal{D}_Q} P(X \rightarrow \tilde{X}) N_B(X \rightarrow \tilde{X}).
 \end{aligned}$$



# Bit error probability



$$k_R = k_A = k_B = -20 \text{ dB}$$

$$\sigma_n^2 = -10 \text{ dB}$$

$$M_R = 4$$

# Nested lattice coding

Lattice:  $\Lambda = \{\mathbf{x} = \mathbf{M}\mathbf{z}, \mathbf{z} \in \mathbb{Z}\}$

$$a\mathbf{x} + b\mathbf{y} \in \Lambda, \text{ for } \mathbf{x}, \mathbf{y} \in \Lambda \text{ and } a, b \in \mathbb{Z}$$

Quantiser:  $Q_{\Lambda}(\mathbf{x}) = \operatorname{argmin}_{\lambda \in \Lambda} \|\mathbf{x} - \lambda\|$

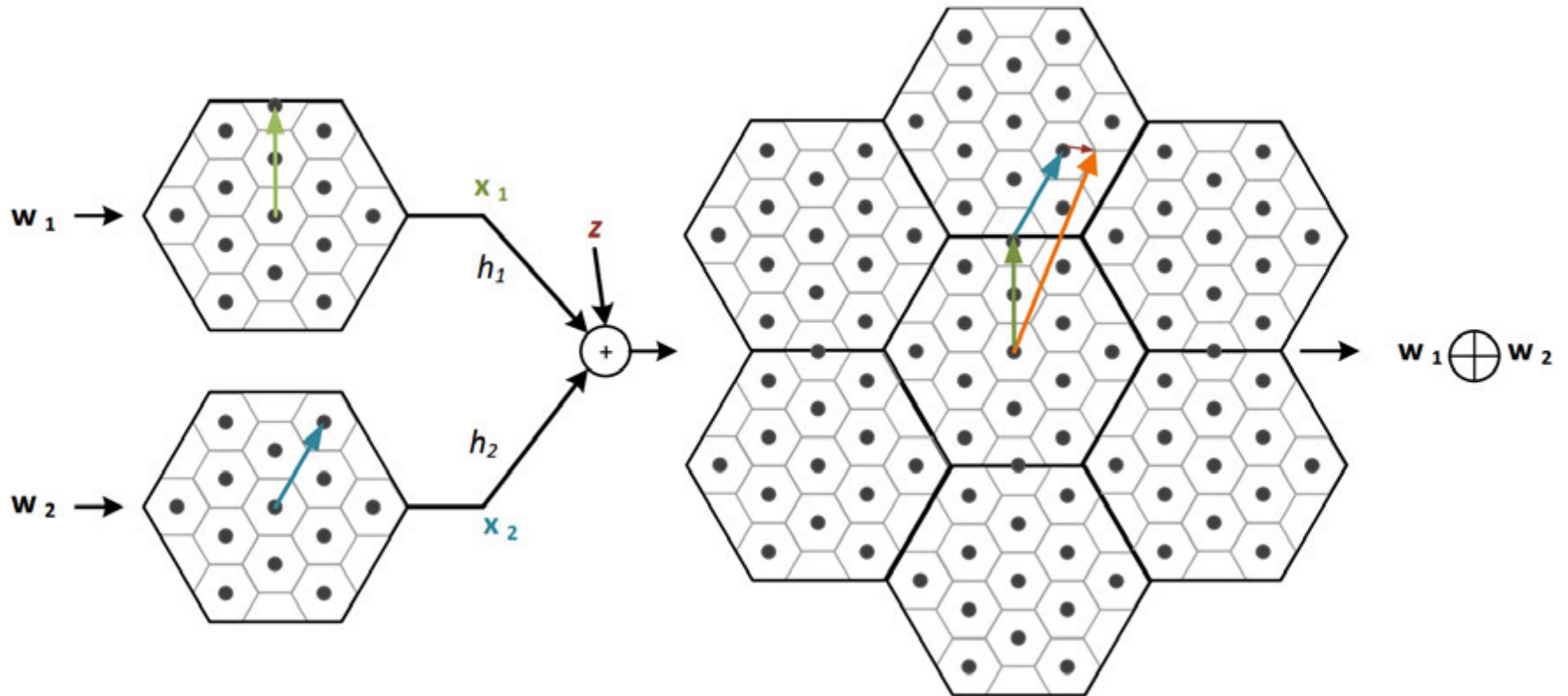
Nested lattice code:

$$\mathcal{L} = \Lambda_F \cap \mathcal{V}_{\Lambda_C}, \in \mathbb{R}^n = \{\lambda = [\lambda_F] \bmod_{\Lambda_C}, \lambda_F \in \Lambda_F\}$$

Isomorphism:  $\phi : \mathbb{F}_Q^n \rightarrow \mathcal{L} = \Lambda_F \cap \mathcal{V}_{\Lambda_C} (\in \mathbb{R}^n)$

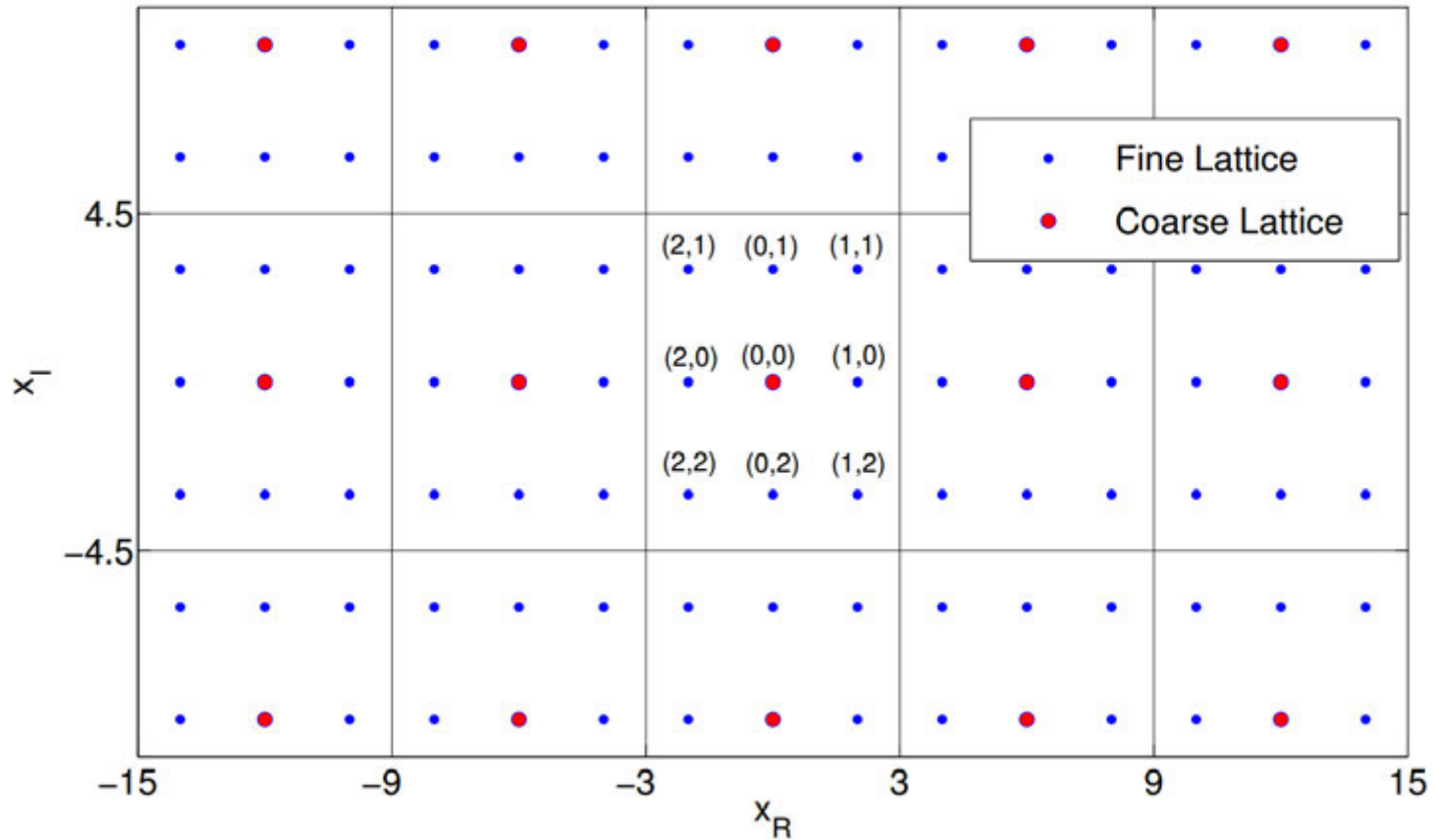
$$S_A, S_B \rightarrow \mathbf{x}_A, \mathbf{x}_B$$

# Lattice-based physical layer NC



# Gaussian lattice

$$\mathbb{Z}_{Q=3}^2 \rightarrow \mathbb{C} : x = \phi(S)$$



## Relay reception

$$\mathbf{y}_R(n) = \sqrt{p_A} \mathbf{h}_{AR} x_A(n) + \sqrt{p_B} \mathbf{h}_{BR} x_B(n) + \sqrt{p_R} k_R \mathbf{H}_{RR} \mathbf{x}_R(n) + \mathbf{n}_R(n)$$

Relay (receiver):  $y_R(n) = h_{AR} \left( \frac{h_{AR}^*}{\langle h_{AR} \rangle} x_A(n) \right) + h_{BR} \left( \frac{h_{BR}^*}{\langle h_{BR} \rangle} x_B(n) \right) + \tilde{n}(n)$

$$= g_{AR} \cdot \phi(S_A) + g_{BR} \cdot \phi(S_B) + \tilde{n}(n)$$

$$g_{AR} = \left( \frac{h_{AR} h_{AR}^*}{\langle h_{AR} \rangle} \right) \quad g_{BR} = \left( \frac{h_{BR} h_{BR}^*}{\langle h_{BR} \rangle} \right)$$

$$v = [a_A \cdot x_A + a_B \cdot x_B] \bmod_{\Lambda_C} \quad x_A, x_B \in \mathcal{L}_G$$

---

### Processing Stage at the Relay for each $y_R(n)$ :

---

- 1) Scale the received relay signal, using the MMSE scaling factor:  $\tilde{y}_R = \alpha \cdot y_R$ ;
  - 2) Quantize  $\tilde{y}_R$  to the closest fine lattice point:  $Q_{\Lambda_F}(\tilde{y}_R)$ ;
  - 3) Perform modulo operation with respect to the coarse lattice to obtain back a point in the nested lattice code:  $x_R = [Q_{\Lambda_F}(\tilde{y}_R)] \bmod_{\Lambda_C}$ .
-



# Terminal's reception

---

## Processing Stage at Terminal $\mathcal{A}$ (similar for $\mathcal{B}$ ):

---

1) Decode the relay transmitted signal, taking into consideration the complex channel effect, using the ML detector for the nested lattice code:  $\hat{x}_R = \arg \min_{\lambda \in \Lambda_F \cap \mathcal{V}_{\Lambda_C}} \| y_A - h_{RA} \lambda \|^2$ ;

2) Map the received information back to the finite field:

First component  $u_1 = \phi^{-1}(\mathcal{R}\{\hat{x}_R\}) = [q_A S_{A,1} + q_B S_{B,1}] \bmod_Q$  and

second component  $u_2 = \phi^{-1}(\mathcal{I}\{\hat{x}_R\}) = [q_A S_{A,2} + q_B S_{B,2}] \bmod_Q$  (where the coefficients  $q_A, q_B$  are naturally given by  $q_A = [a_A] \bmod_Q$  and  $q_B = [a_B] \bmod_Q$ );

3) Subtract own information:  $w_{A,1} = [u_1 - q_A S_{A,1}] \bmod_Q = q_B S_{B,1}$  and

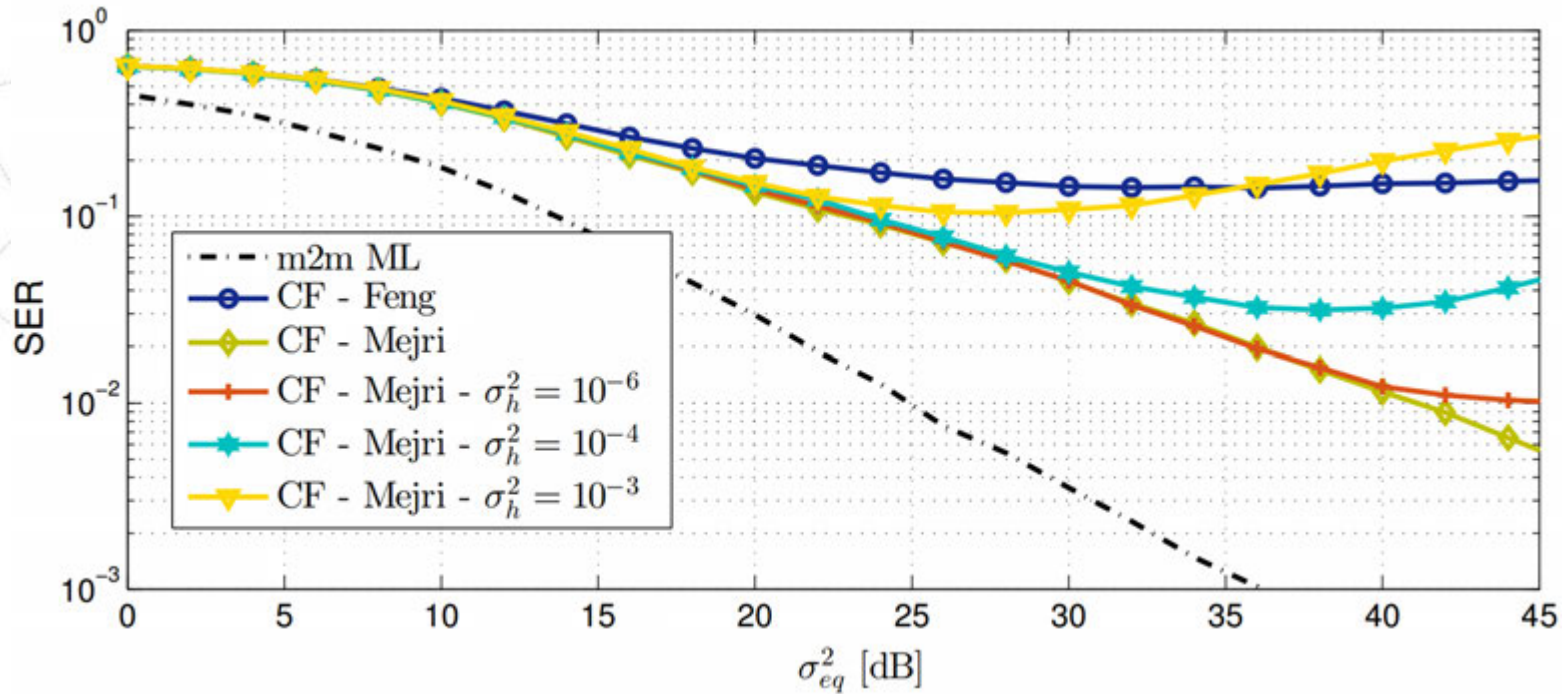
$w_{A,2} = [u_2 - q_A S_{A,2}] \bmod_Q = q_B S_{B,2}$ ;

4) Remove channel integer effect over the finite field,  $q_B$ , to obtain  $\hat{S}_B = (\hat{S}_{B,1}; \hat{S}_{B,2})$ .

---

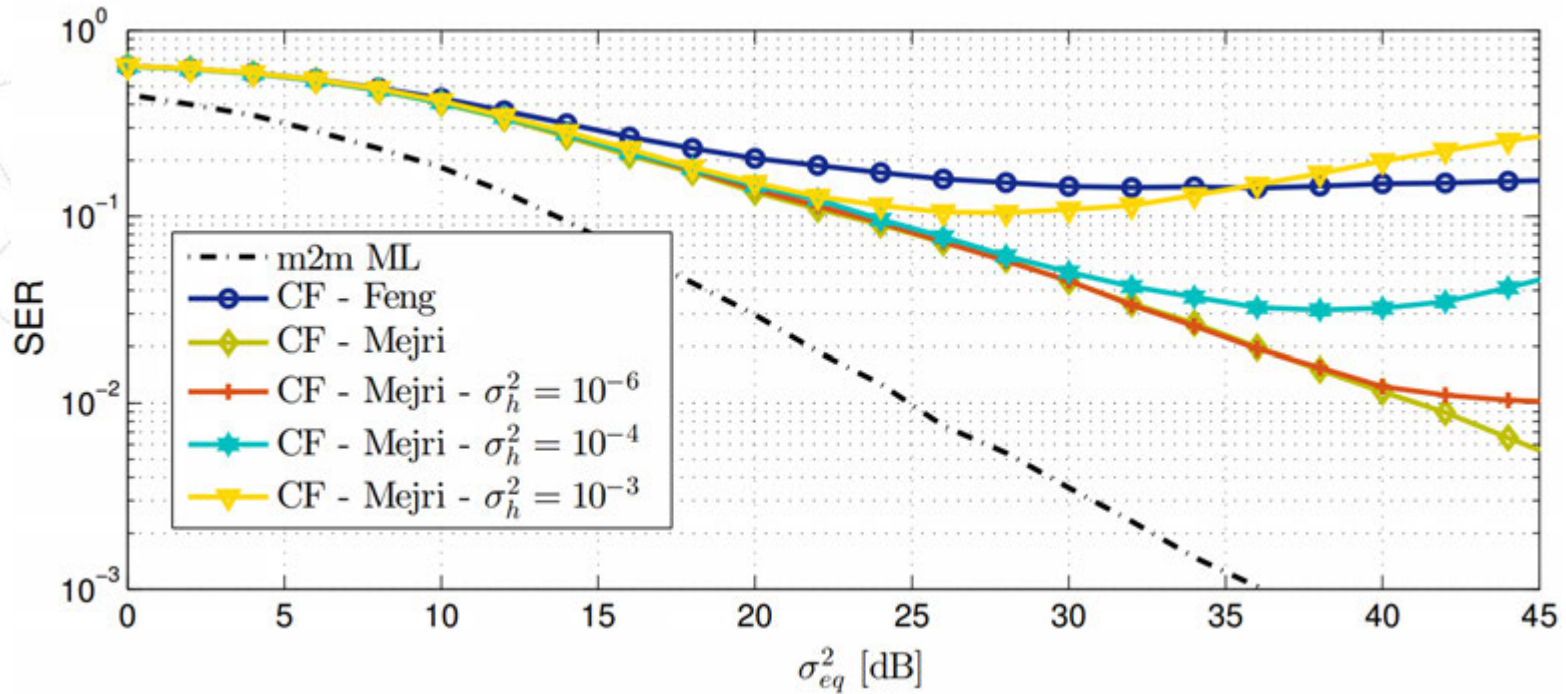


# TWRC with lattice-based PLNC



MMSE scale factor: 
$$\alpha = \frac{\gamma \cdot \mathbf{g} \mathbf{a}^H}{1 + \gamma \cdot \mathbf{g} \mathbf{g}^H}$$

# TWRC with lattice-based PLNC



From an engineering perspective the scheme has a very poor performance.

How to improve it?

## Equivalent noise is composed of:

Cross-terms interference:

$$(\mathbf{H}_{BR}^\dagger \mathbf{H}_{AR} \mathbf{x}_A(n) + \mathbf{H}_{AR}^\dagger \mathbf{H}_{BR} \mathbf{x}_B(n))$$

Self-interference:

$$(\mathbf{H}_{BR}^\dagger + \mathbf{H}_{AR}^\dagger) k_R \mathbf{H}_{RR} \mathbf{x}_R(n)$$

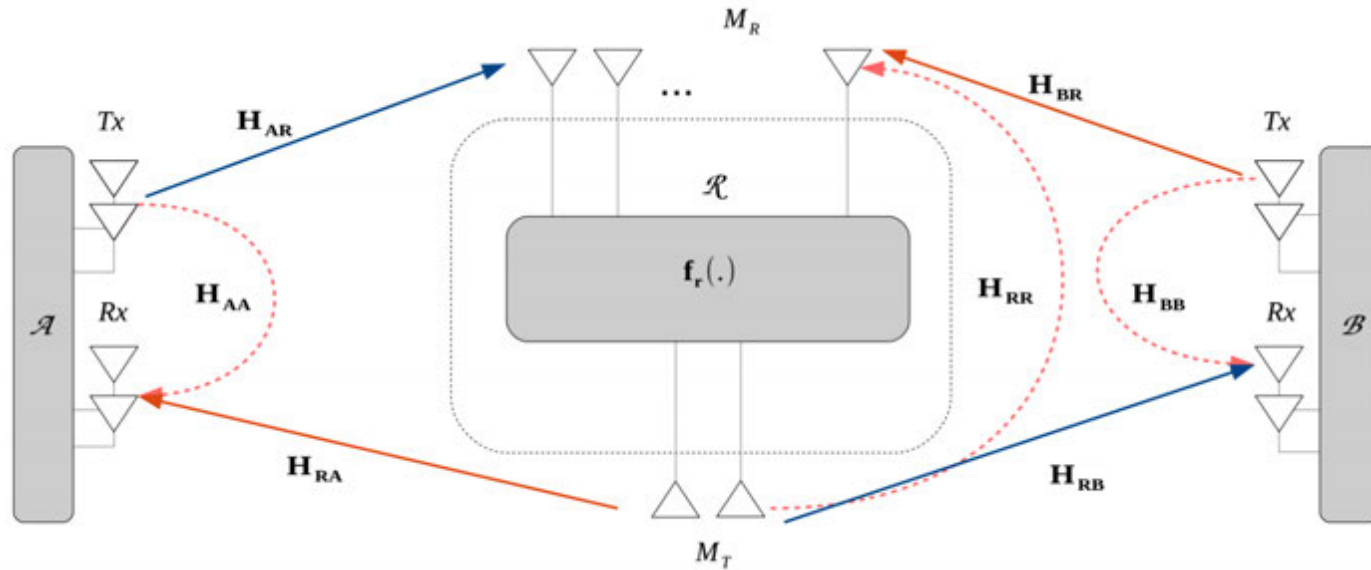
Noise term:

$$(\mathbf{H}_{BR}^\dagger + \mathbf{H}_{AR}^\dagger) \mathbf{n}_R(n)$$

But, given the massive MIMO effect:

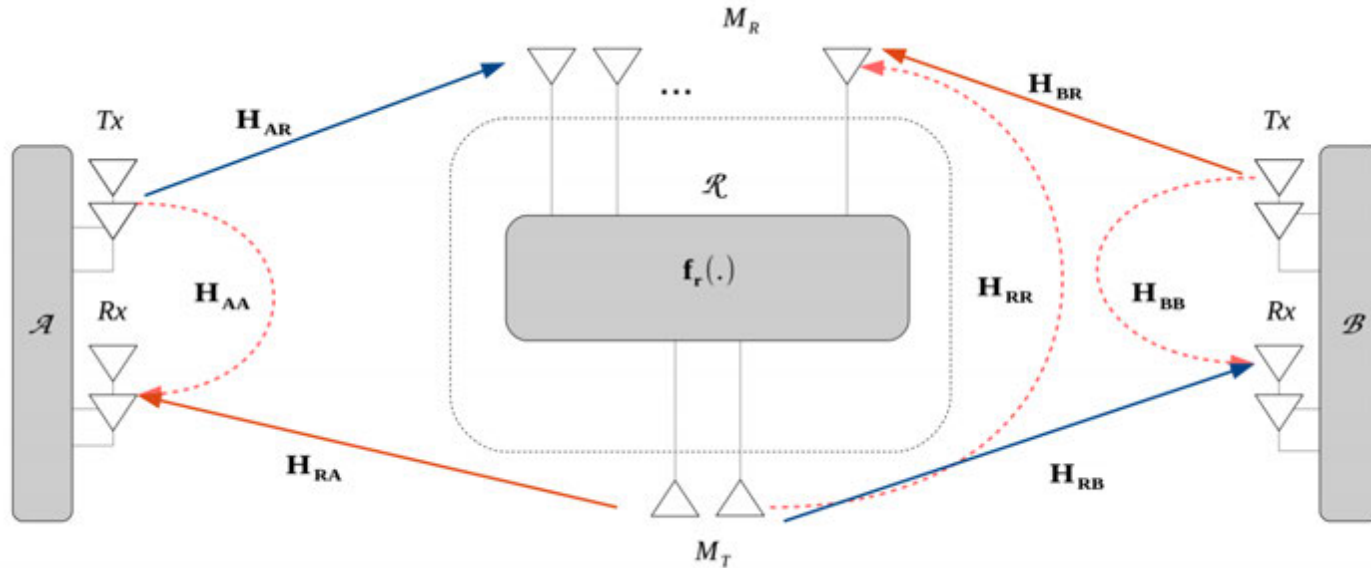
$$\mathbf{H}_{BR}^\dagger \mathbf{H}_{AR} \rightarrow \mathbf{0}, \mathbf{H}_{AR}^\dagger \mathbf{H}_{BR} \rightarrow \mathbf{0}, \text{ as } M_R \rightarrow \infty$$

# Taking advantage of massive MIMO



$$\begin{aligned} \mathbf{y}_R(n) &= \mathbf{H}_{AR}\mathbf{x}_A(n) + \mathbf{H}_{BR}\mathbf{x}_B(n) + k_R\mathbf{H}_{RR}\mathbf{x}_R(n) + \mathbf{n}_R(n) \\ &= \mathbf{H}_{AR}\mathbf{x}_A(n) + \mathbf{H}_{BR}\mathbf{x}_B(n) + \tilde{\mathbf{n}}_R(n) \end{aligned}$$

# Taking advantage of massive MIMO

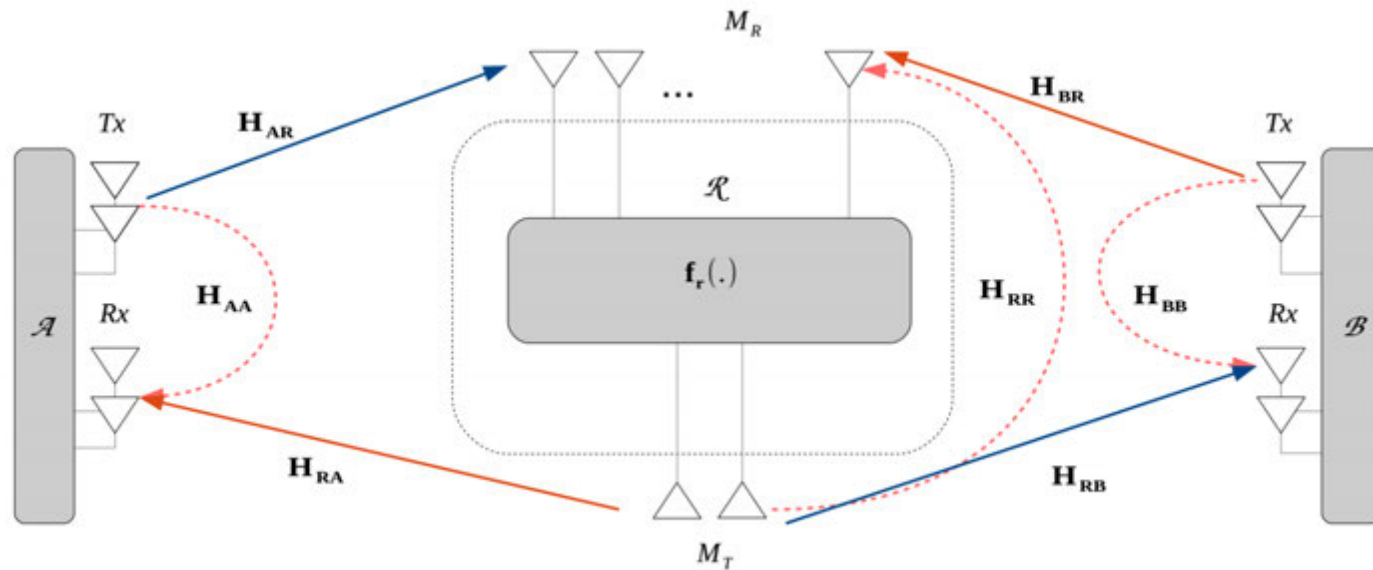


$$\begin{aligned}
 \mathbf{y}_P(n) &= \mathbf{H}_{AR}^T \mathbf{y}_R(n) + \mathbf{H}_{BR}^T \mathbf{y}_R(n) \\
 &= (\mathbf{H}_{AR}^H \mathbf{H}_{AR})^{-1} \mathbf{H}_{AR}^H \mathbf{y}_R(n) + (\mathbf{H}_{BR}^H \mathbf{H}_{BR})^{-1} \mathbf{H}_{BR}^H \mathbf{y}_R(n) \\
 &= (\mathbf{H}_{AR}^\dagger \mathbf{H}_{AR} \mathbf{x}_A(n) + \mathbf{H}_{BR}^\dagger \mathbf{H}_{BR} \mathbf{x}_B(n)) \\
 &\quad + (\mathbf{H}_{BR}^\dagger \mathbf{H}_{AR} \mathbf{x}_A(n) + \mathbf{H}_{AR}^\dagger \mathbf{H}_{BR} \mathbf{x}_B(n)) + (\mathbf{H}_{BR}^\dagger + \mathbf{H}_{AR}^\dagger) \mathbf{n}_R(n) \\
 &= \underbrace{\mathbf{D}_A \mathbf{x}_A(n) + \mathbf{D}_B \mathbf{x}_B(n)}_{\text{desired component}} + \underbrace{\tilde{\mathbf{n}}_R^*(n)}_{\text{equivalent total noise}}
 \end{aligned}$$

$$\left[ \mathbf{D}_A \mathbf{x}_A(n) + \mathbf{D}_B \mathbf{x}_B(n) \right] \text{mod}_{\Lambda_C}$$



# Reception at both terminals



$$y_A(n) = \mathbf{H}_{RA}x_R(n) + \tilde{n}_A(n)$$

$$y_B(n) = \mathbf{H}_{RB}x_R(n)$$



# Reception at the relay and at both terminals

---

## Procedure 3 PLNC Scheme for Massive MIMO Relaying

---

### Processing stage at the relay for each $y_R(n)$ :

1) Zero forcing process of the received signal:  $y_P(n) = \mathbf{H}_{AR}^\dagger y_R(n) + \mathbf{H}_{BR}^\dagger y_R(n)$ ;

**for**  $i = 1, \dots, N_T$  **do**

2) Scale the processed relay signal, using the MMSE scaling factor  $y_{P,i}$ :  $\tilde{y}_{P,i} = \alpha_i \cdot y_{P,i}$ ;

3) Quantize  $\tilde{y}_{P,i}$  to the closest fine lattice point:  $Q_{\Lambda_F}(\tilde{y}_{P,i})$ ;

4) Perform modulo operation with respect to the coarse lattice to obtain back a point of the nested lattice code:  $x_{R,i} = [Q_{\Lambda_F}(\tilde{y}_{P,i})] \bmod_{\Lambda_C}$ ;

**end for**

5) Transmit the signal  $\mathbf{x}_R(n) = [x_{R,1}, \dots, x_{R,N_T}]$ ;

### Processing stage at terminal $\mathcal{A}$ (similar for $\mathcal{B}$ ):

1) Decode the relay transmitted signal using the ML detector for the nested lattice code:

$$\hat{\mathbf{x}}_R = \arg \min_{\lambda \in (\Lambda_F \cap \mathcal{V}_{\Lambda_C})^{N_T}} \|\mathbf{y}_A - \mathbf{H}_{RA} \lambda\|;$$

**for**  $i = 1, \dots, N_T$  **do**

2) Map the received information back to the finite field:  $u_{1,i} = \phi^{-1}(\mathcal{R}\{\hat{x}_{R,i}\}) = [S_{A,1,i} + S_{B,1,i}] \bmod Q$   
and  $u_{2,i} = \phi^{-1}(\mathcal{I}\{\hat{x}_{R,i}\}) = [S_{A,2,i} + S_{B,2,i}] \bmod Q$  (where here  $q_A = 1, q_B = 1$ );

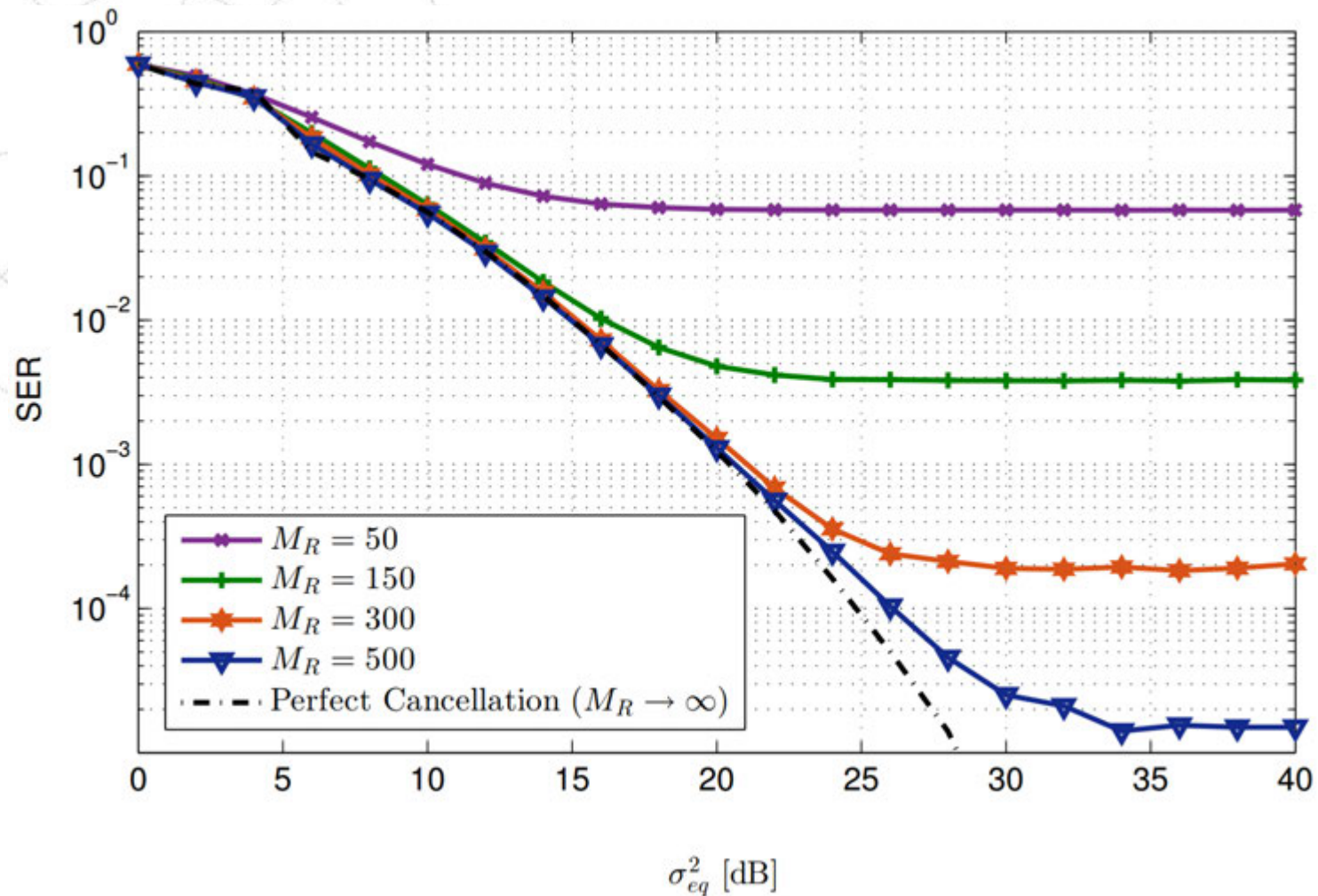
3) Subtract own information to obtain:  $\hat{S}_{B,1,i} = [u_{1,i} - S_{A,1,i}] \bmod Q$  and  $\hat{S}_{B,2,i} = [u_{2,i} - S_{A,2,i}] \bmod Q$ ;

**end for**

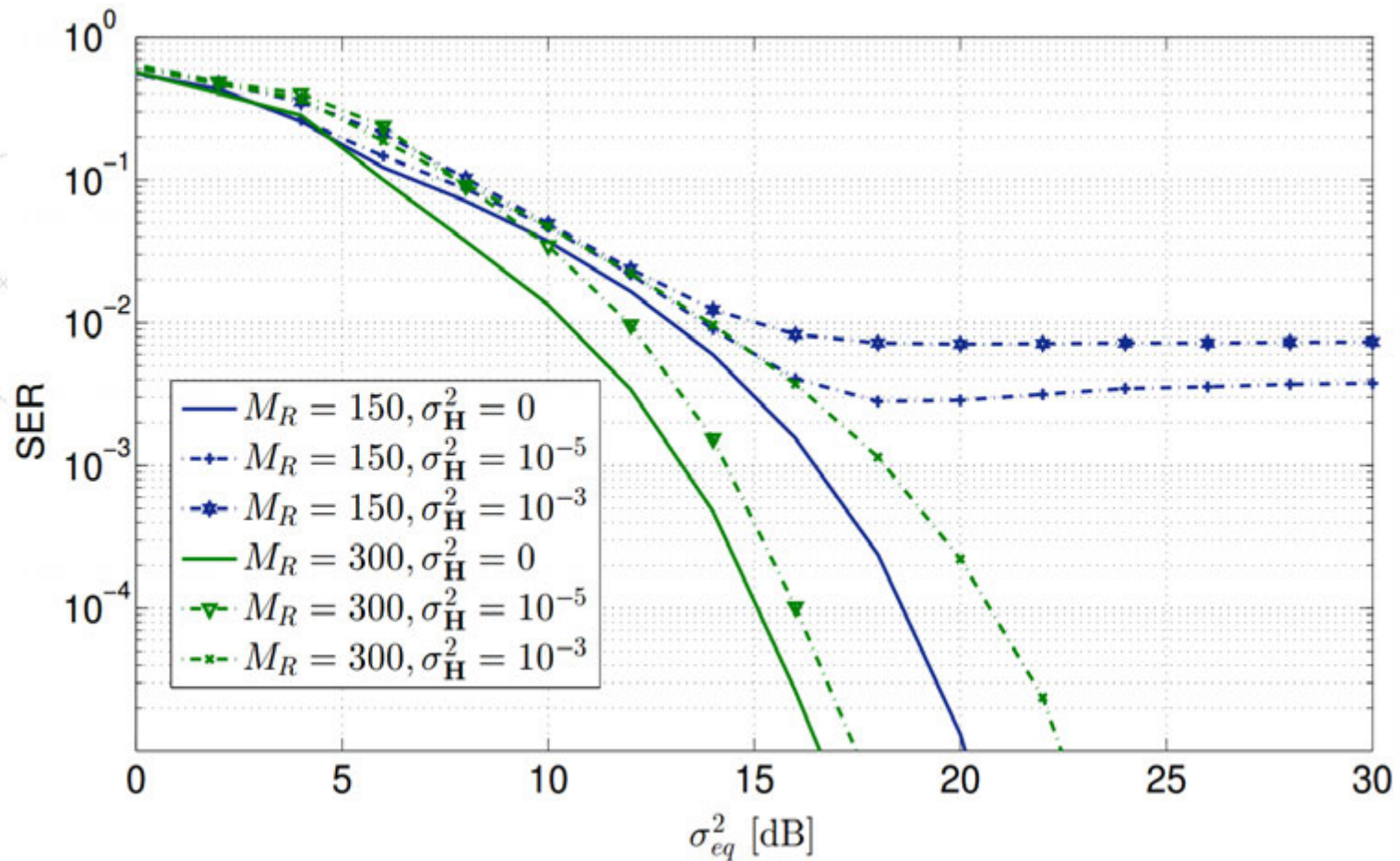
4) Obtain  $\hat{S}_{B,i} = (\hat{S}_{B,1,i}; \hat{S}_{B,2,i})$  for  $i = 1, \dots, N_T$ .

---

# Results: massive MIMO + full-duplex + PLNC

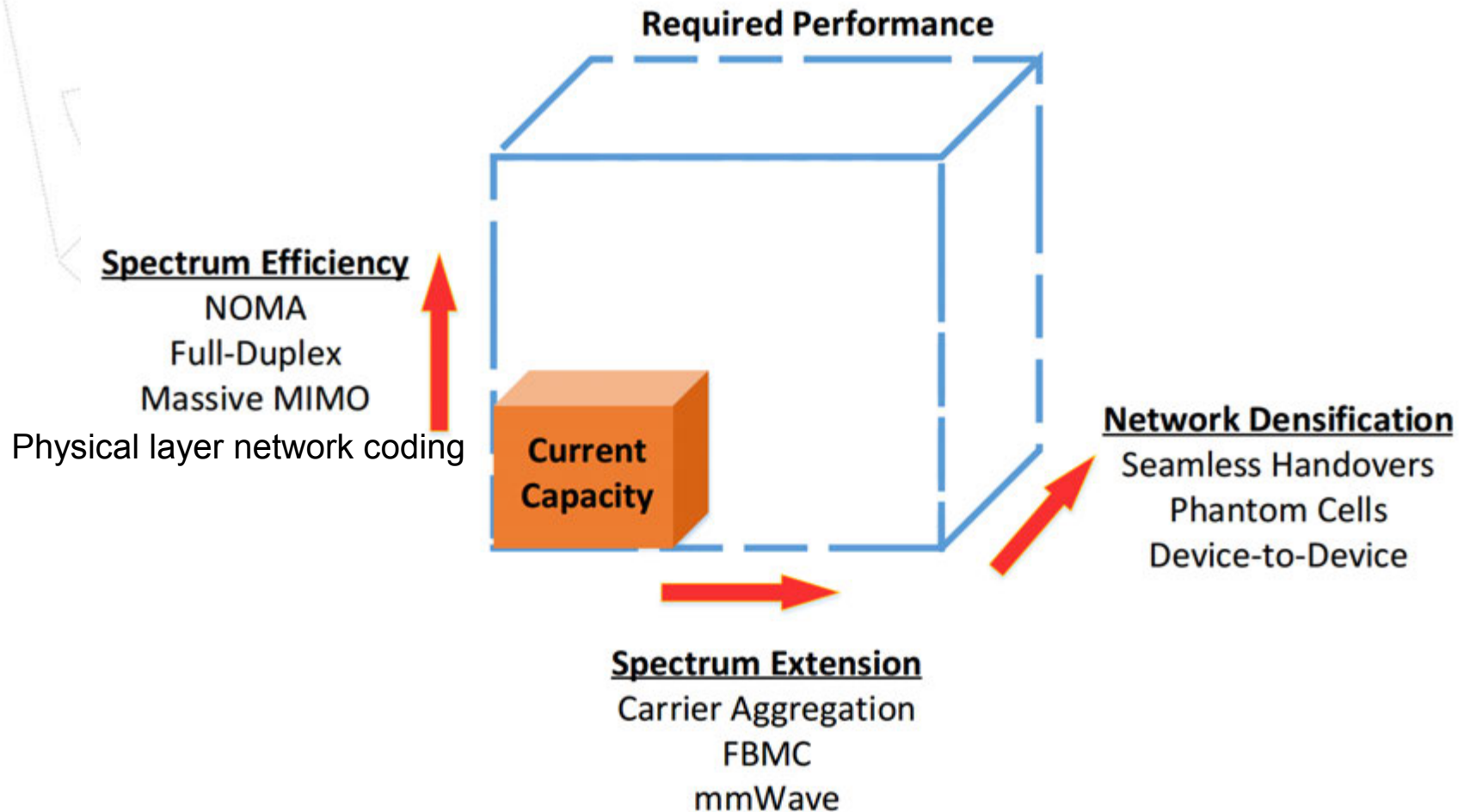


# Effect of channel estimation errors



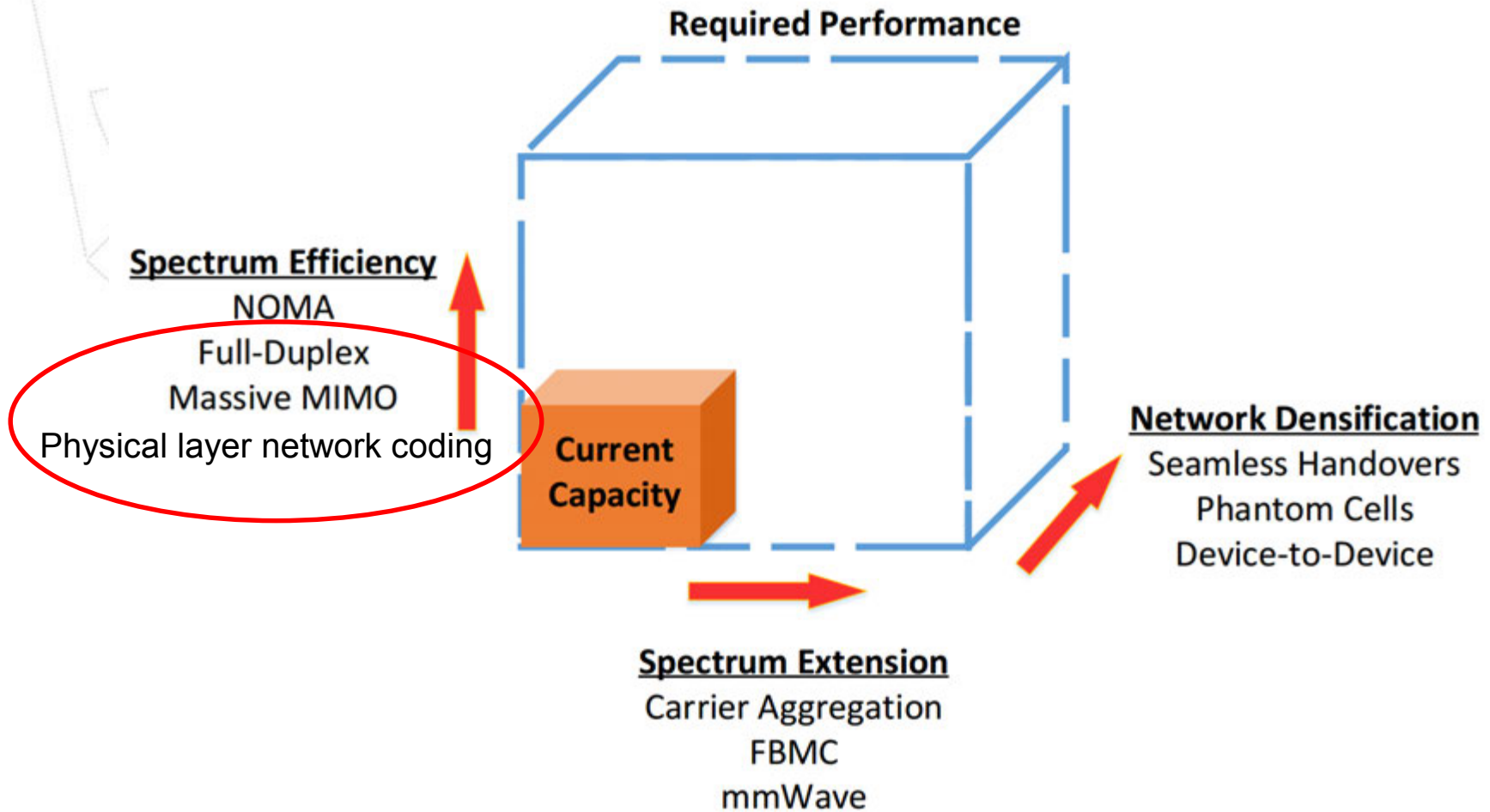
Channel state information accuracy is crucial !

# How to offer more?





# How to offer more?



# Acknowledgements

Funding:

# FCT

Fundação para a Ciência e a Tecnologia

 instituto de  
telecomunicações

Graduate Students:

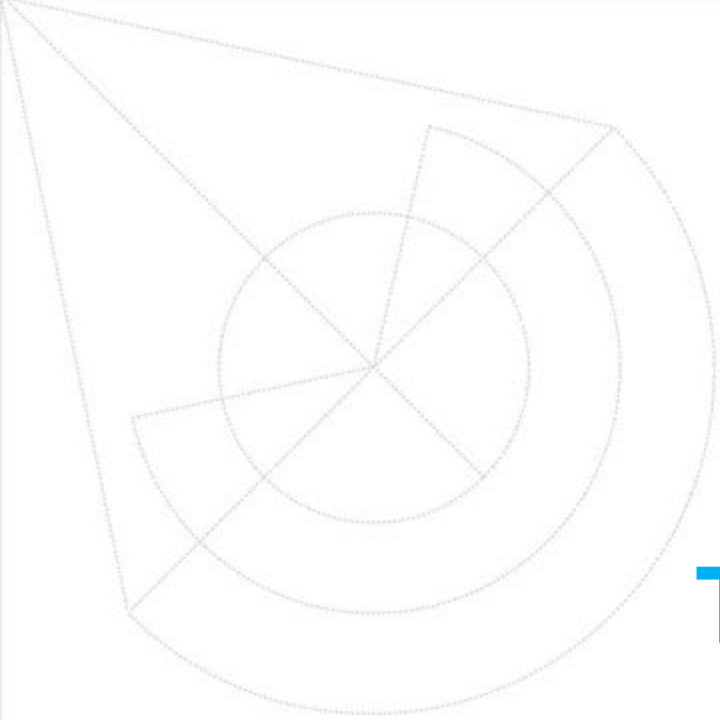


Francisco Rosário



João Sande Lemos





# Thank you



**Back up slides**

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- A. Vardy, "Trellis Structure of Codes," in Handbook of Coding Theory, Vera Pless and W. Cary Huffman, Eds. Amsterdam, The Netherlands: Elsevier, 1998, ch. 24, pp. 1989-2117.

# “Lattice are everywhere\*”

## Current growing/hot topics:

- Physical Layer Network Coding

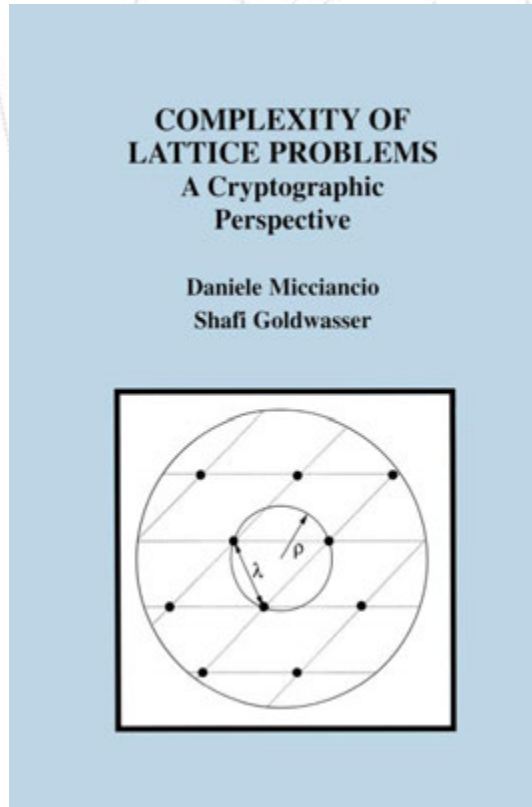
[Gastpar, Nazer, Proc. of the IEEE, 2011];

- Lattice-based cryptography for physical layer security.

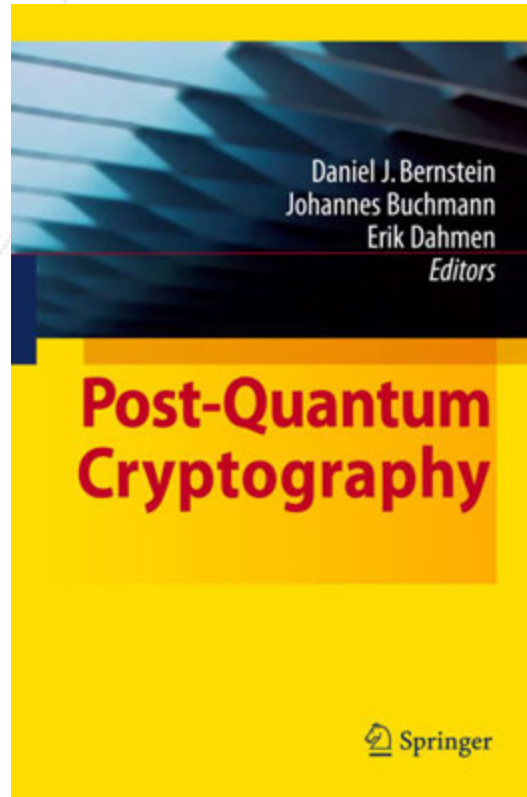
• R. Zamir, “Lattices are Everywhere”, *talk at the Information Theory and Applications Workshop (ITA09)*, University of California at San Diego, February 2009.

- U. Erez, S. Litsyn and R. Zamir, “Lattices which are good for (almost) everything”, *IEEE Transactions on Information Theory*, pp. 3401-3416 Oct. 2005.

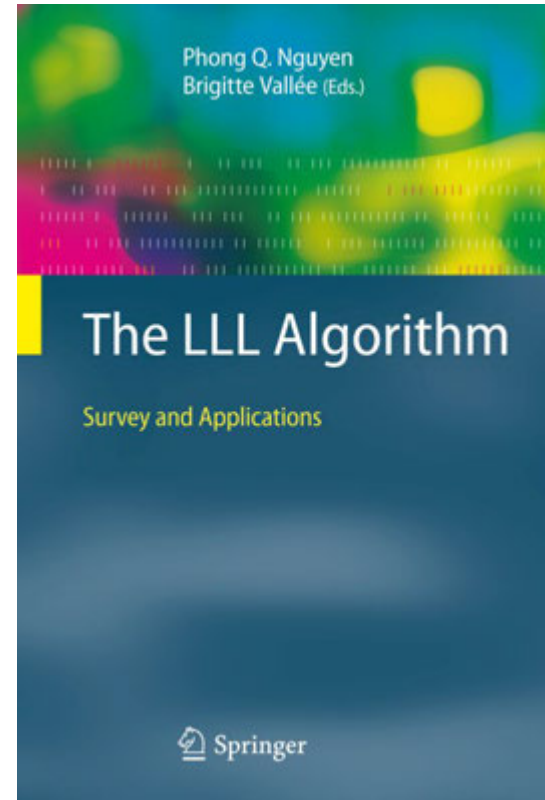
# Lattices in Cryptography



2002



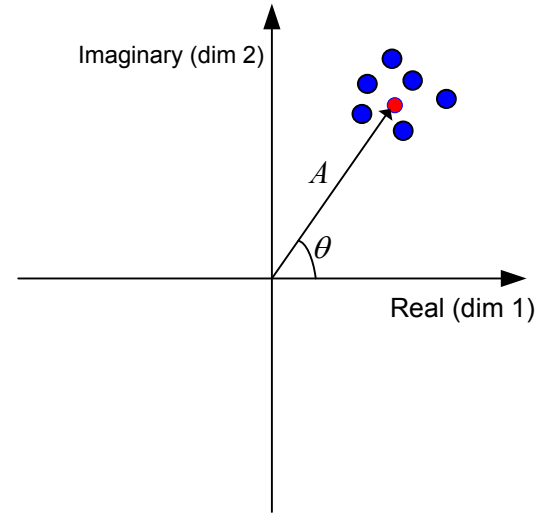
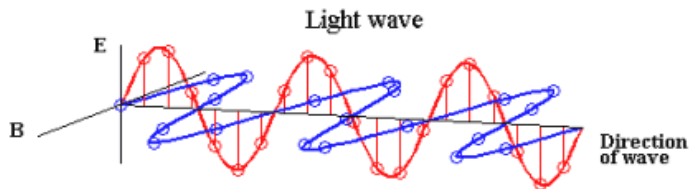
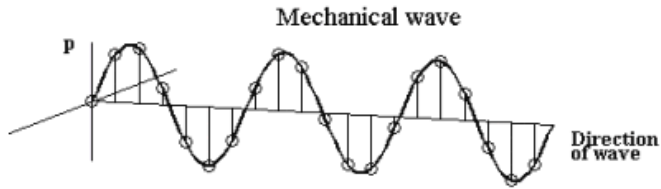
2009



2009

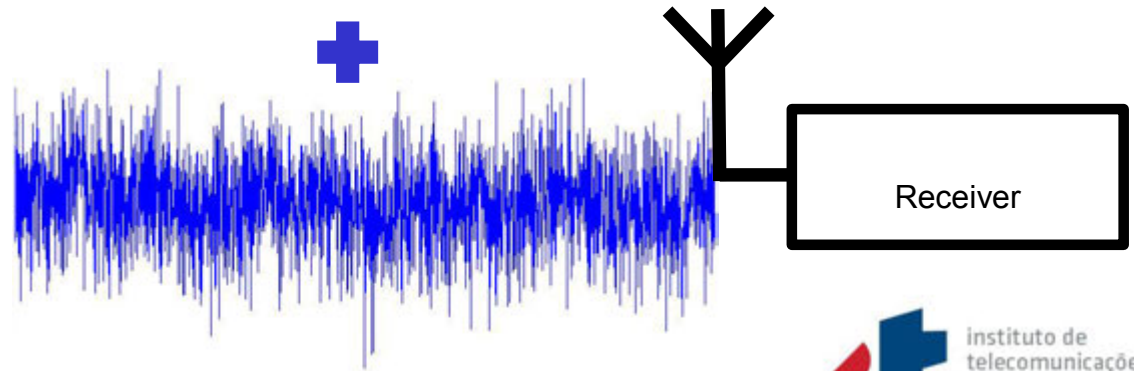


# Radio waves as symbols



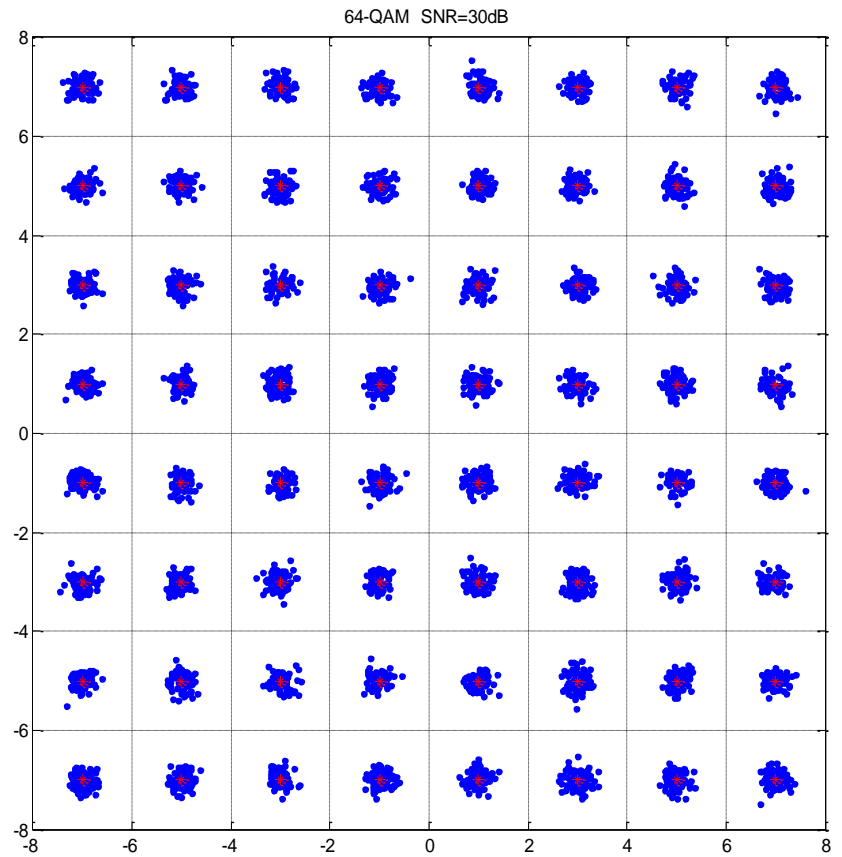
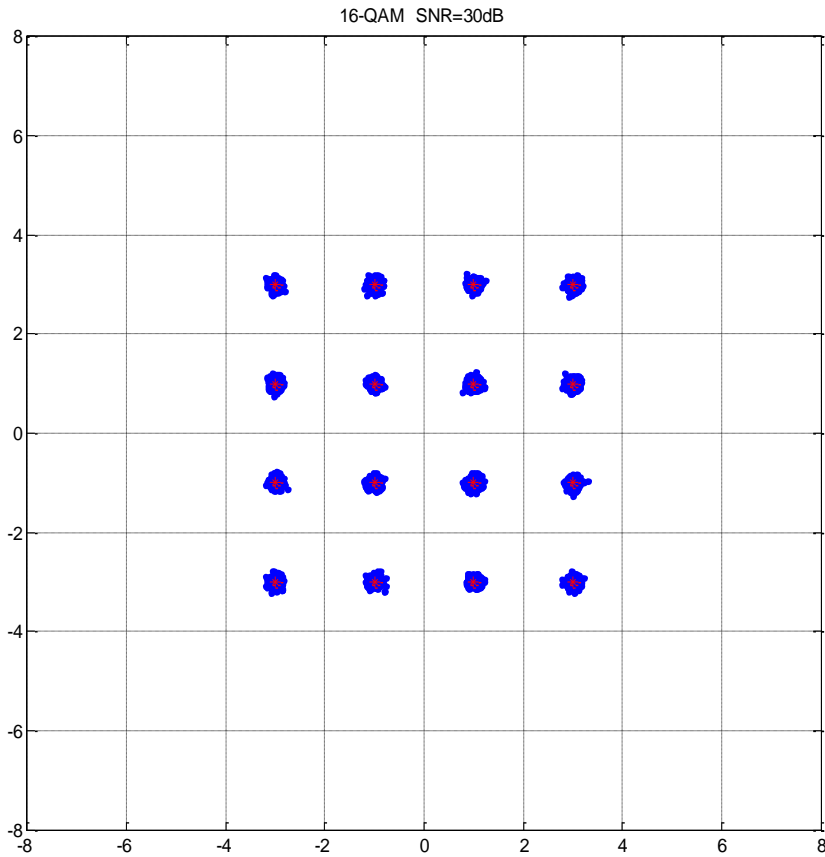
*isvr*

[Picture from the Institute of Sound and Vibration, University of Southampton, UK]



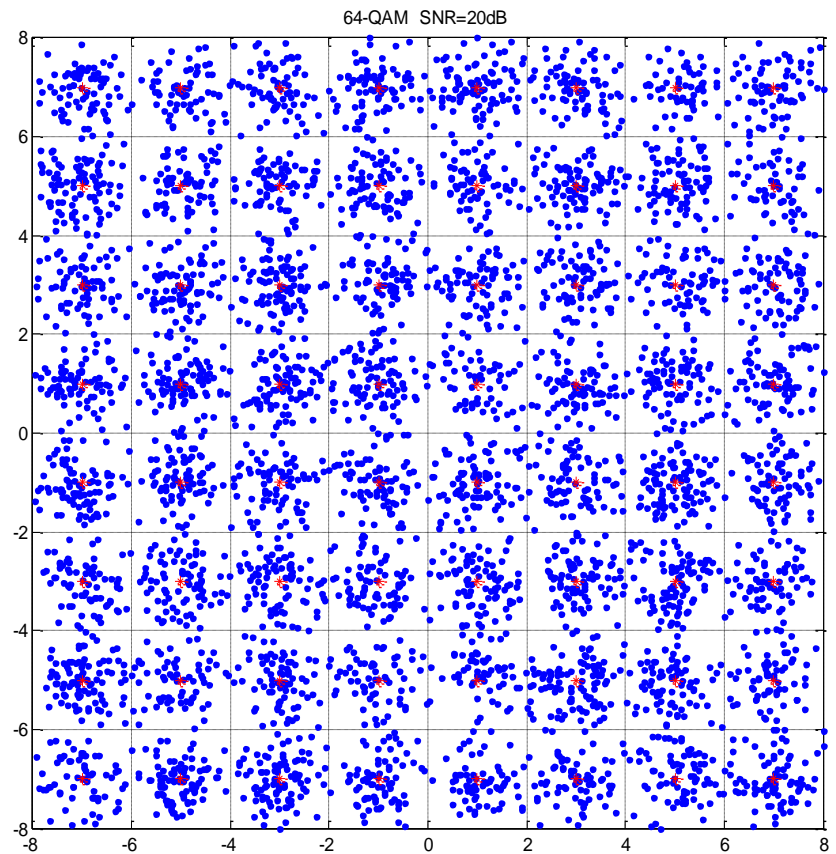
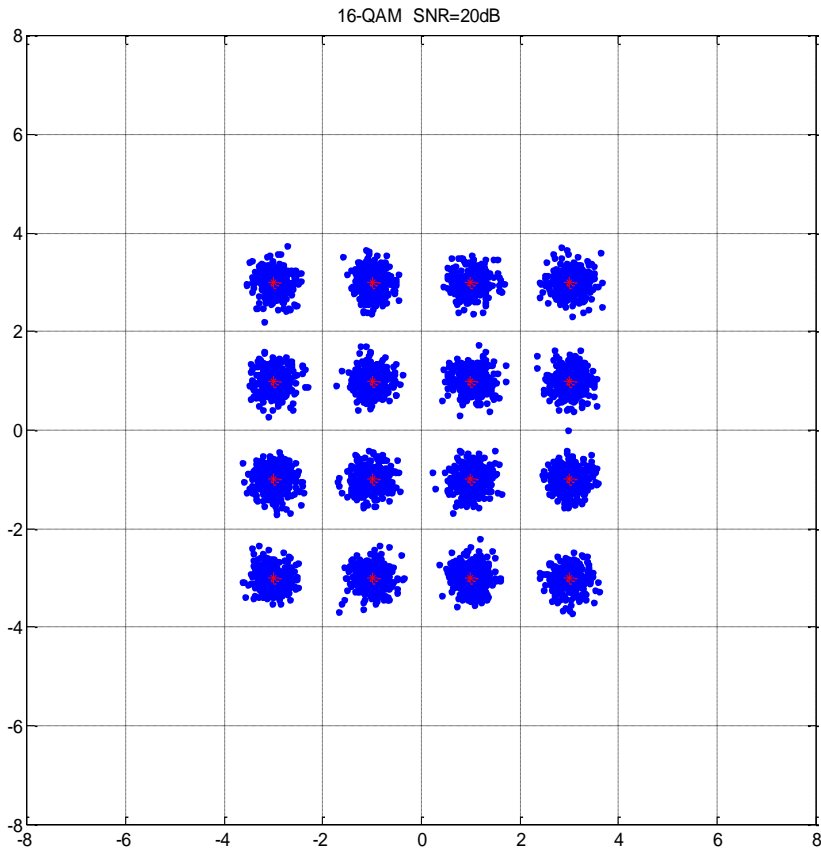
# 2D orthogonal lattices in SISO

30 dB



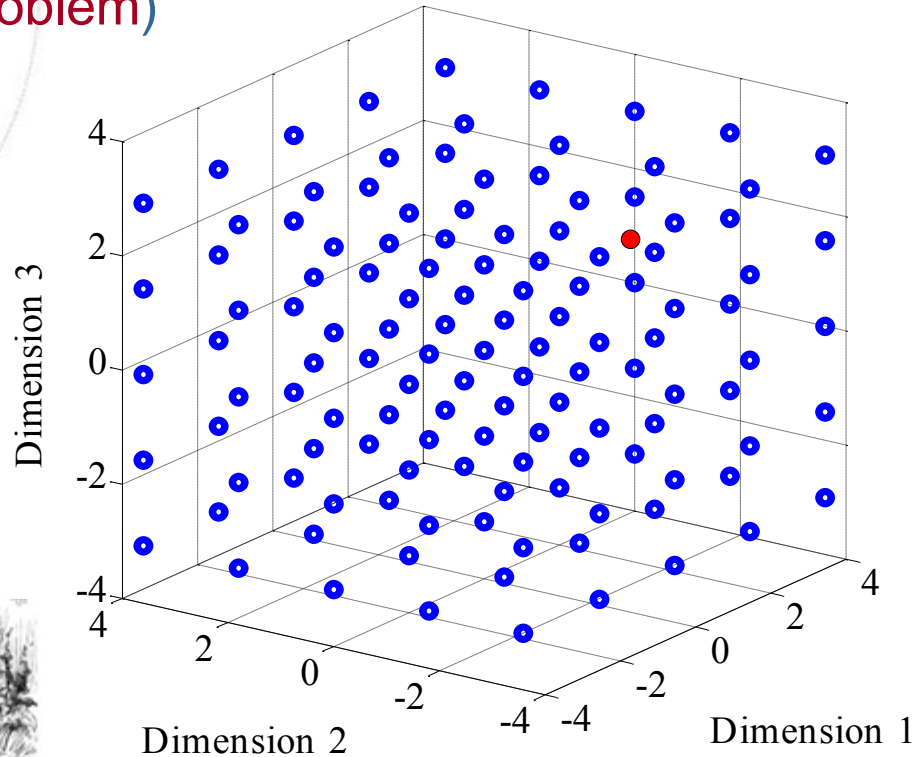
# 2D orthogonal lattices in SISO

20 dB



# CVP in lattices in 8 dimensions (or more)

Closest vector problem (CVP) in a lattice  
(NP-hard problem)

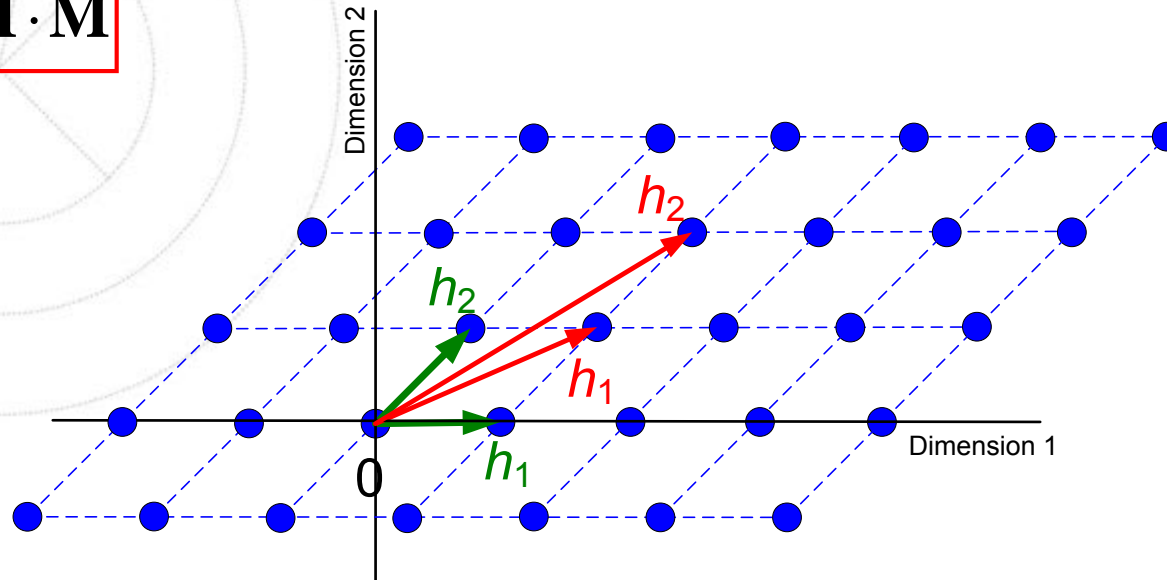


Closest point to  
the red one?



# Equivalent basis: reduced and not reduced

$$\mathbf{H}_{eq} = \mathbf{Q} \cdot \mathbf{H} \cdot \mathbf{M}$$



- Almost orthogonal vectors
- Short vectors

# LLL reduction

- Reduction in polynomial time Lenstra, Lenstra, Lovász (1982)
- The Gauss algorithm (1801) is LLL in 2D



Arjen Klaas  
Lenstra

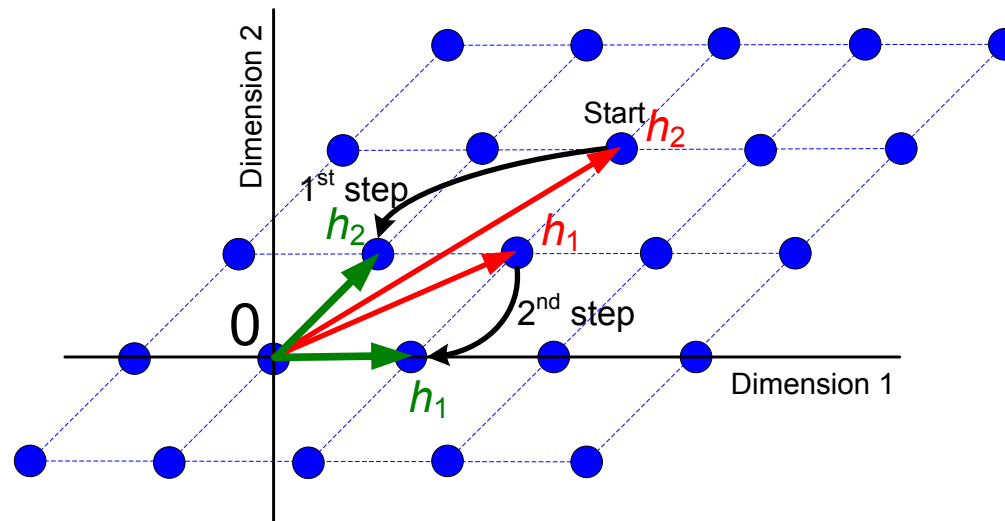


Hendrik Willem  
Lenstra



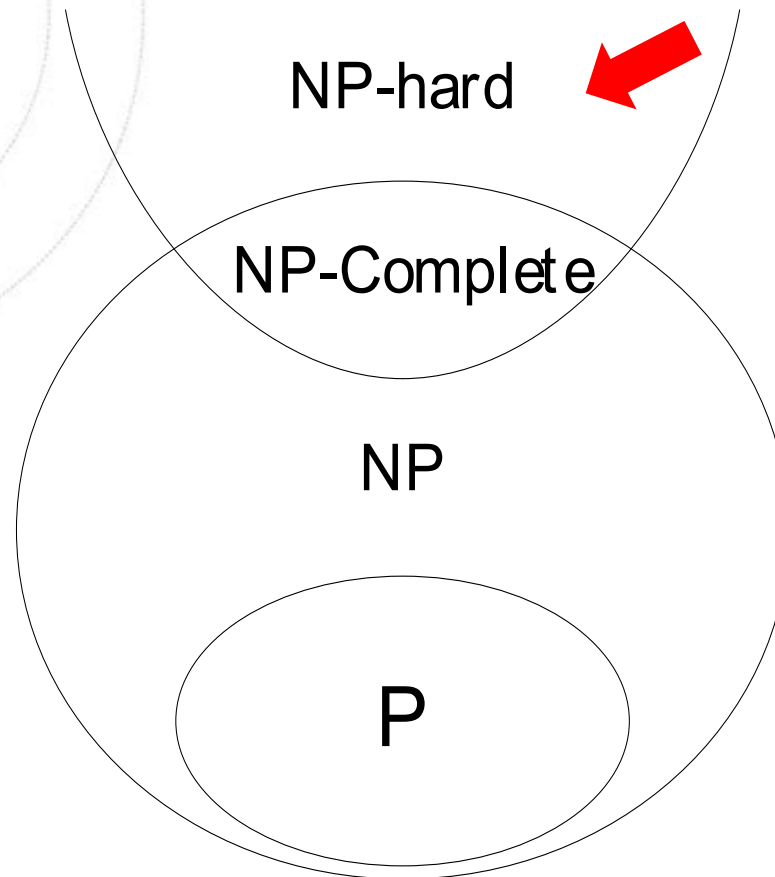
László Lovász

1<sup>st</sup> step: replace  $h_2$  by  $h_2 - h_1$  which gives us  $h_2$   
2<sup>st</sup> step: replace  $h_1$  by  $h_1 - h_2$  which gives us  $h_1$





# NP Hard Problems: the worst

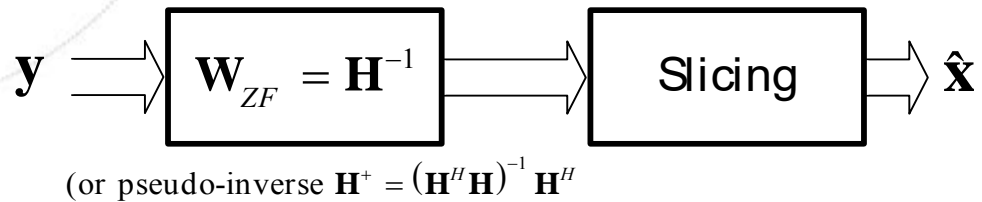


# Linear receivers

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

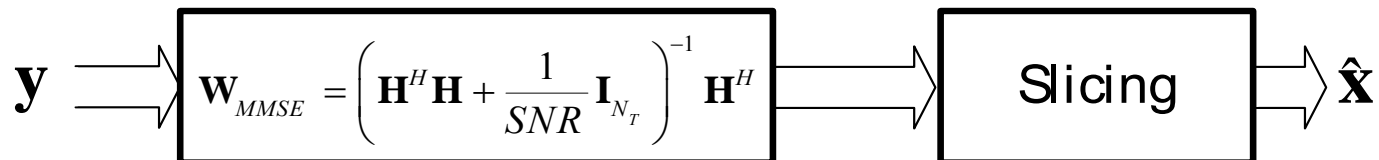
Zero Forcing (ZF)

$$\hat{\mathbf{x}} = \text{slice}(\mathbf{H}^{-1}\mathbf{y})$$

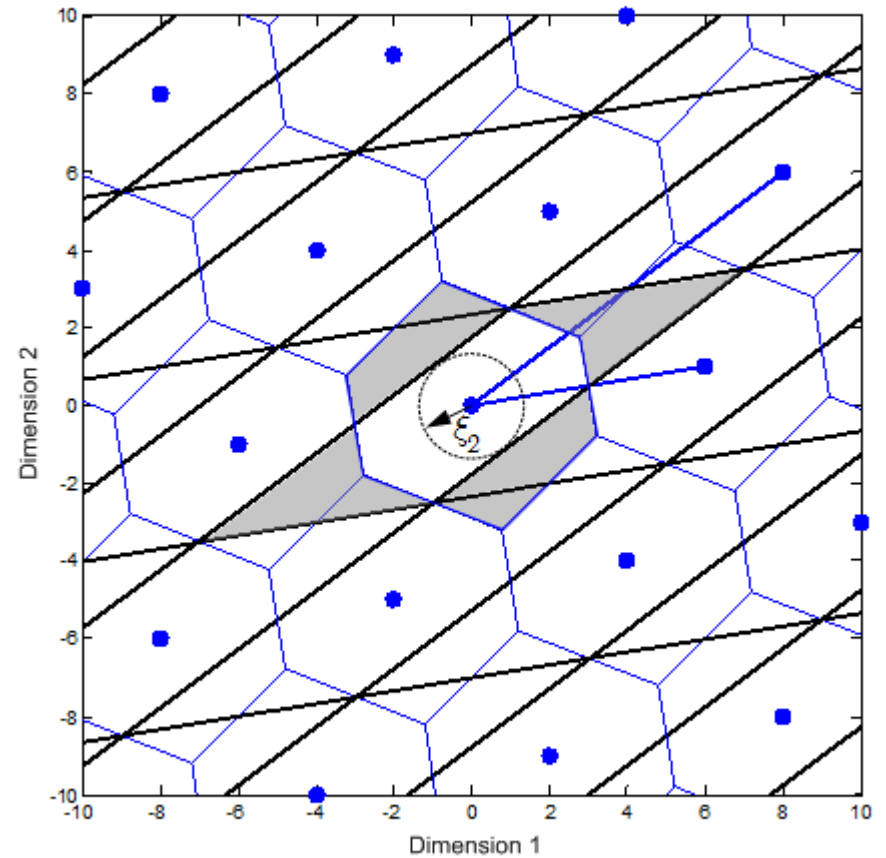
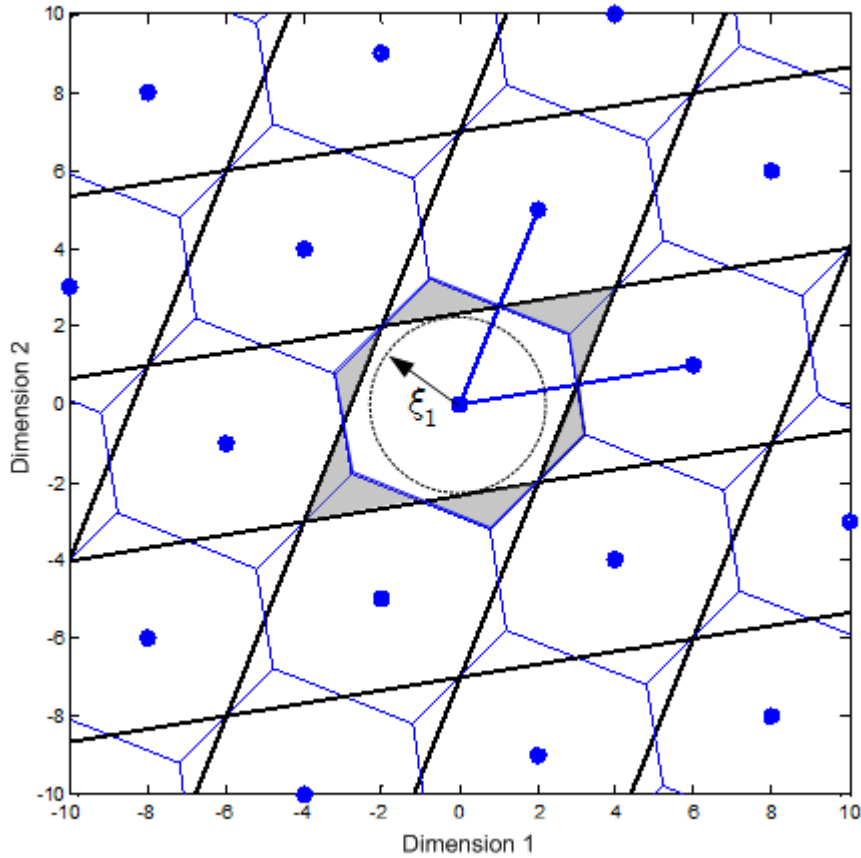


Minimum Mean Squared Error (MMSE)

$$E \left[ \|\mathbf{W}\mathbf{y} - \mathbf{x}\|^2 \right]$$



# The geometry of zero-forcing

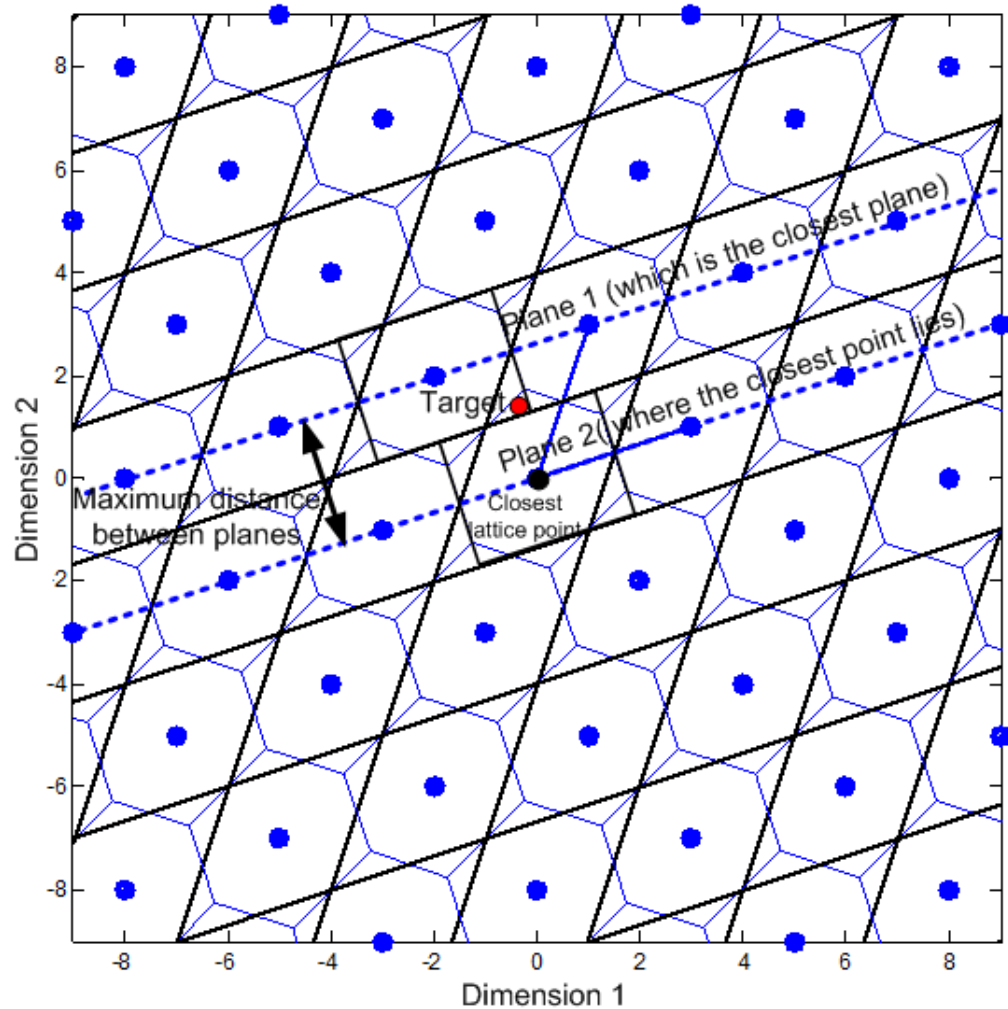


# The geometry of successive interference cancelation

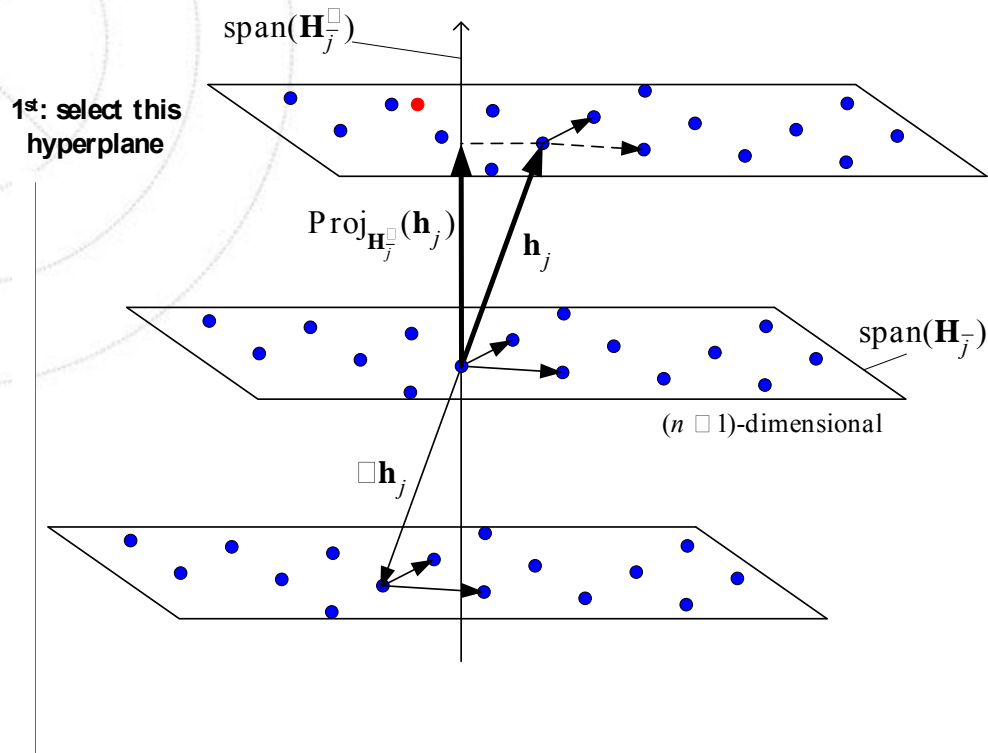


$$\mathbf{H} = \mathbf{QR}$$

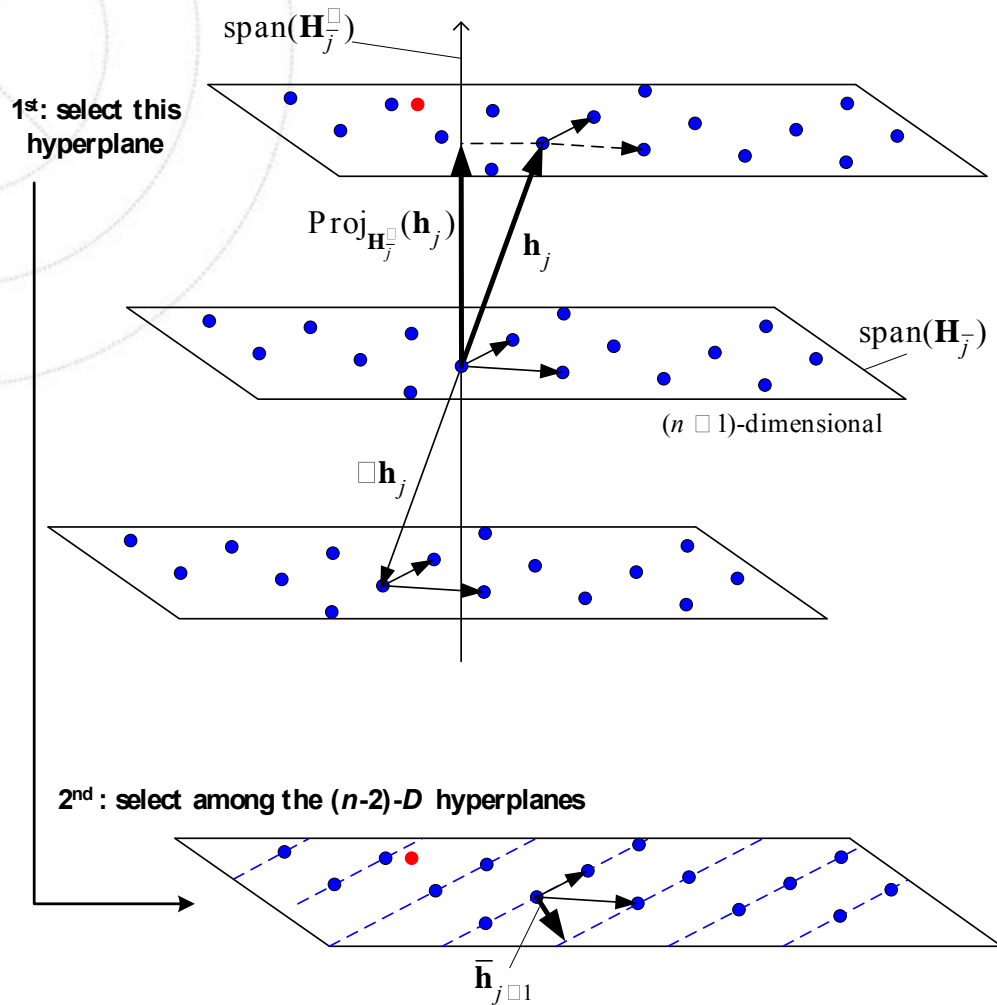
$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \end{pmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \mathbf{n}_3 \end{bmatrix}$$



V-BLAST - vertical Bell Labs space-time [1999]  
 or... SIC: successive interference cancelation  
 or... Babai's algorithm or the nearest plane algorithm [1986]

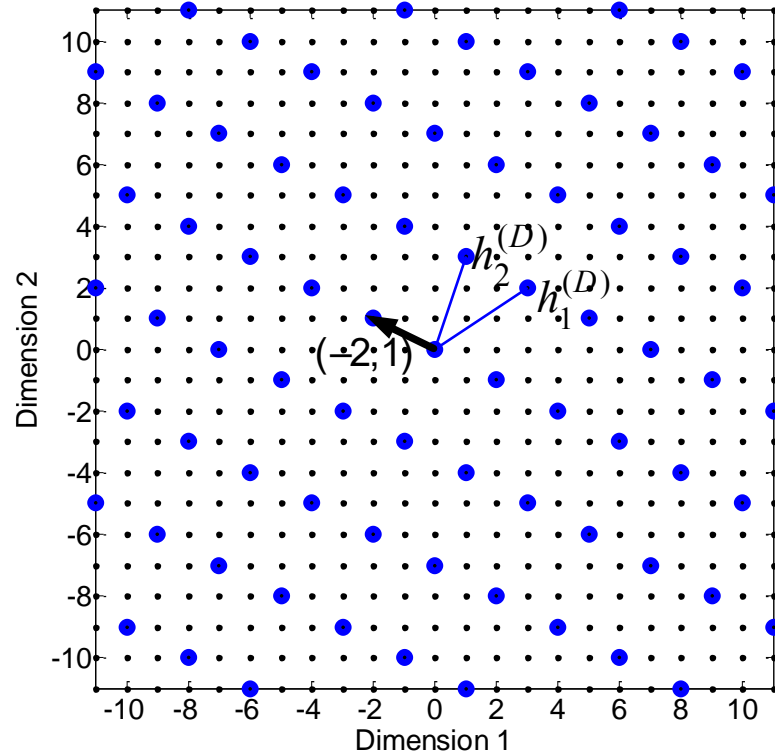
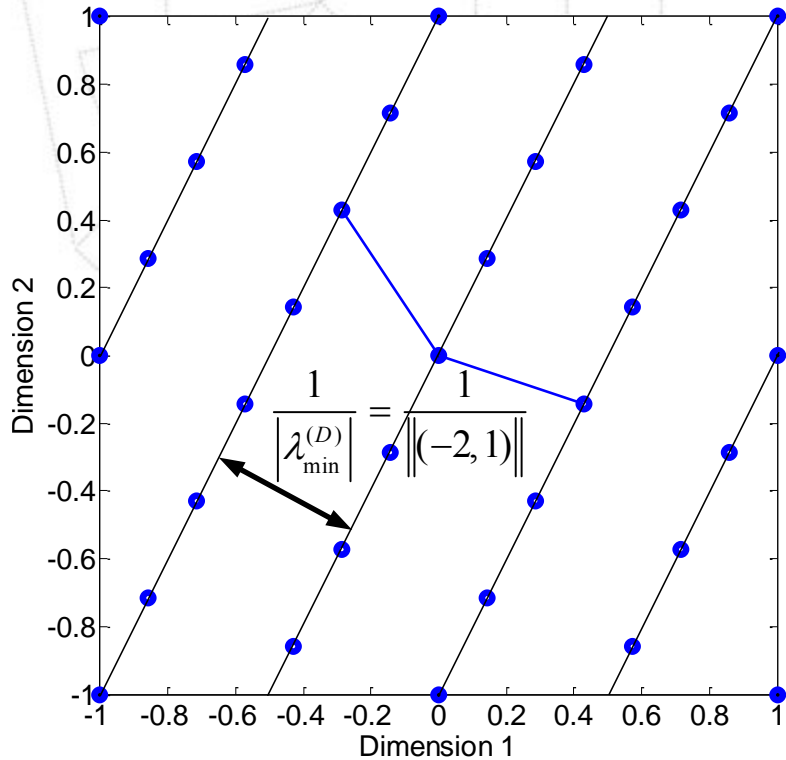


V-BLAST - vertical Bell Labs space-time [1999]  
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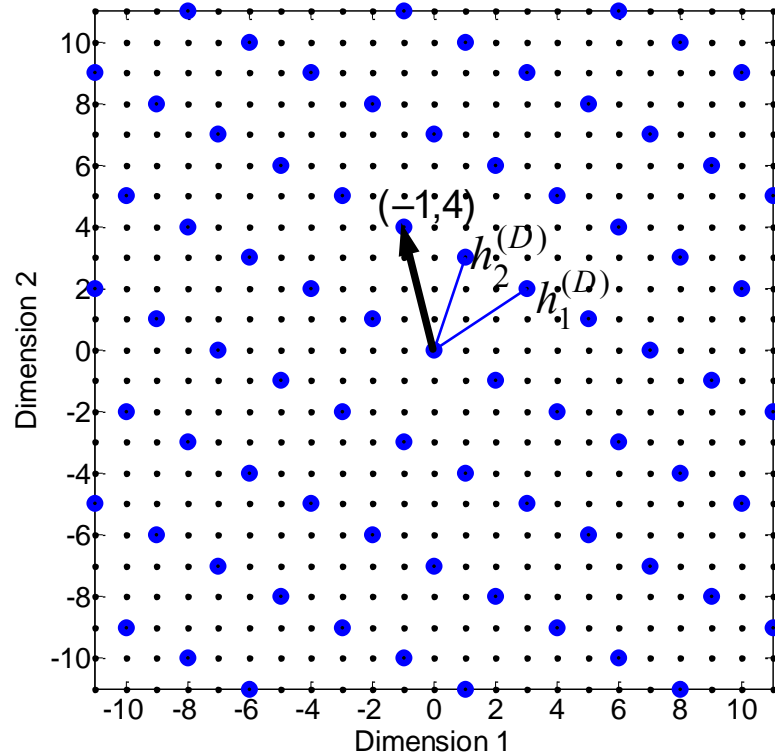
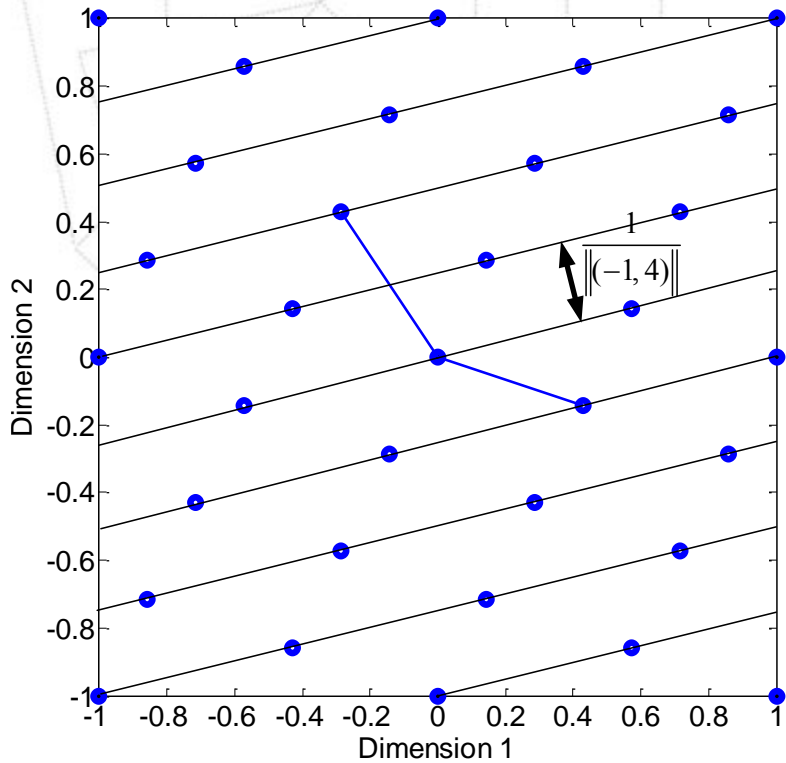


# Vectors in the Dual Lattice define hyperplanes (1/2)



(a) Selection of  $(-2,1)$  in the dual lattice.

# Vectors in the Dual Lattice define hyperplanes (2/2)

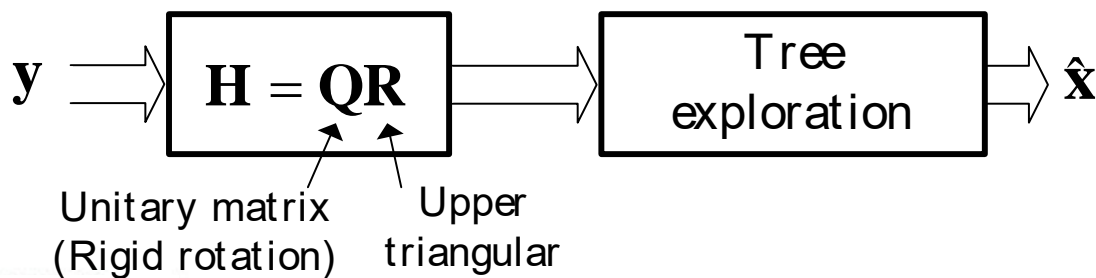
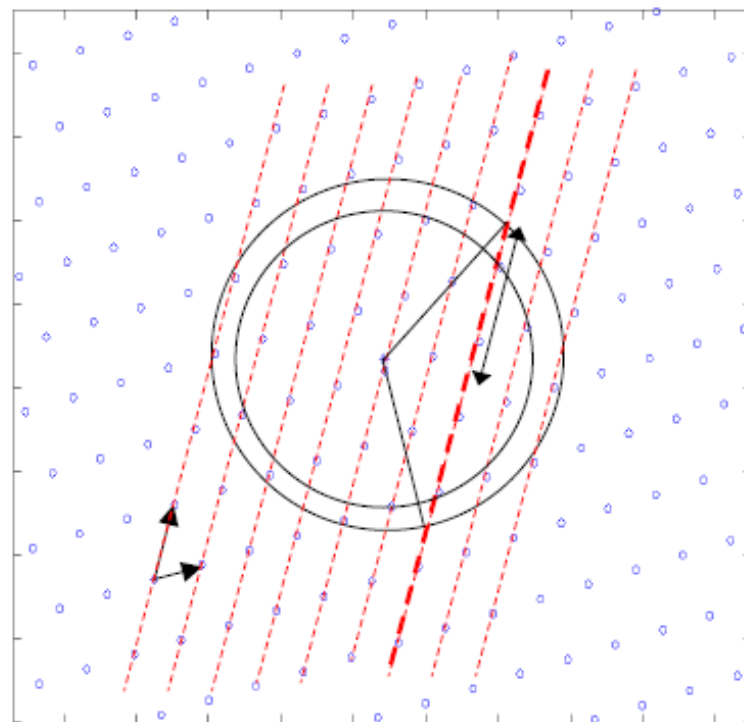


(b) Selection of  $(-1, 4)$  in the dual lattice.

# The geometry of sphere decoding

$$\| \mathbf{y} - \mathbf{H}\mathbf{x} \|^2 \leq \epsilon$$

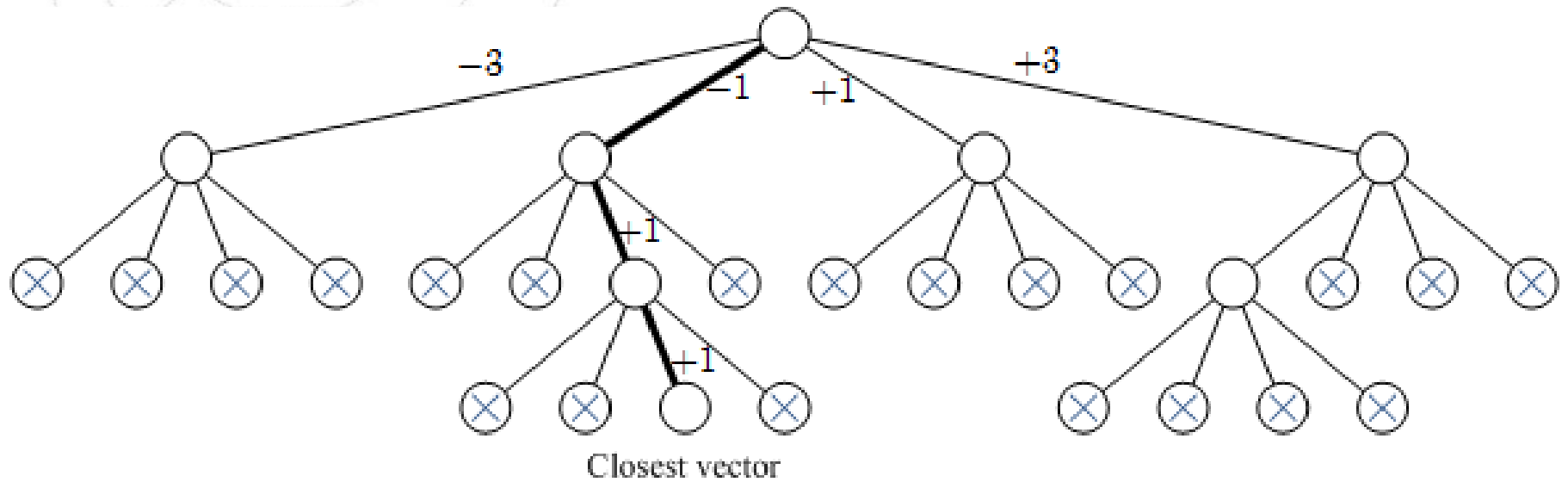
$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \end{pmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \mathbf{n}_3 \end{bmatrix}$$



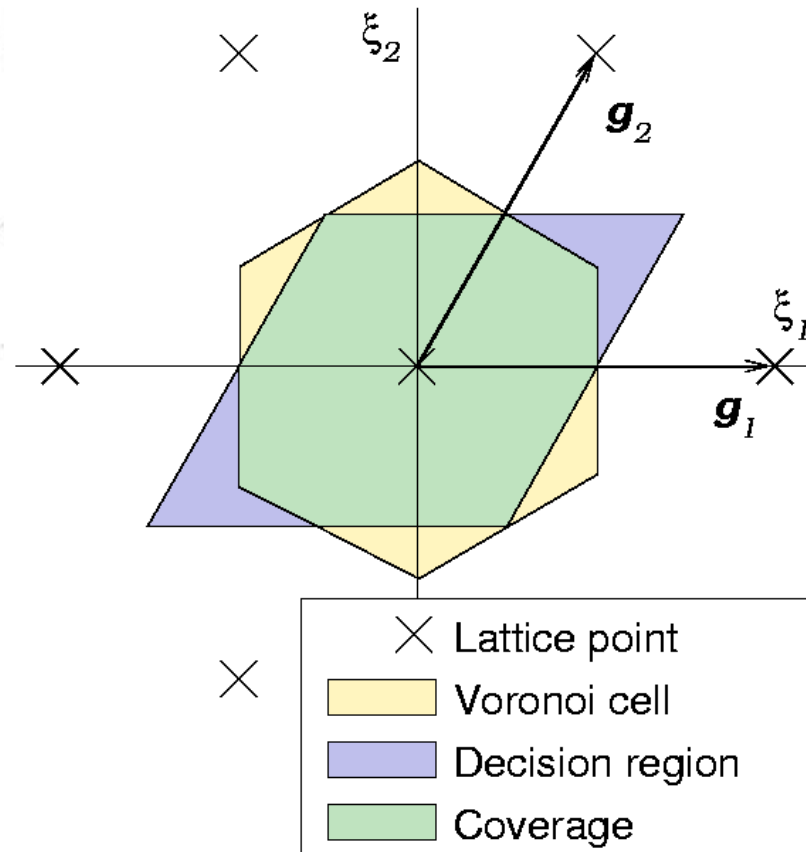
[Figure by Dr. Wai Ho Mow, Univ. of hong Kong]

$$\sum_{i=1}^m \hat{a}_i^2 - \sum_{j=i}^m r_{ij}^2 x(j)^2 \leq \epsilon$$

# Sphere decoding or... tree decoding



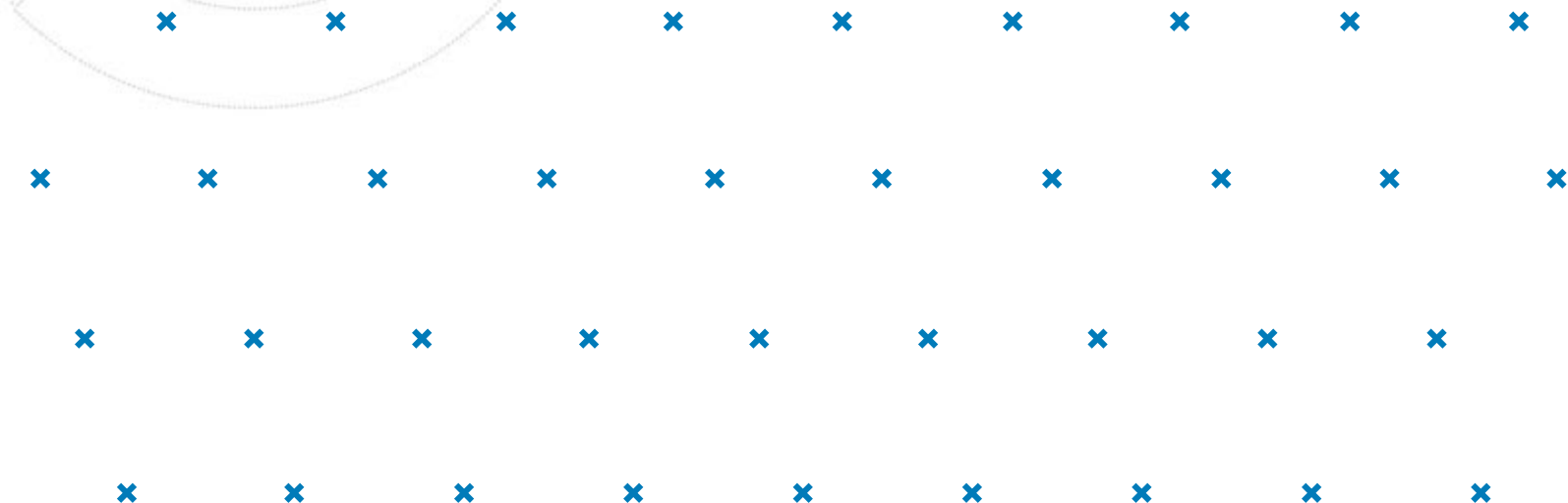
# The notion of coverage (1/2)



[Figure by Dr. Karen Su, University of Cambridge]

# The notion of coverage (2/2)

Improves coverage of Voronoi cell by increasing the inradius of the decision region

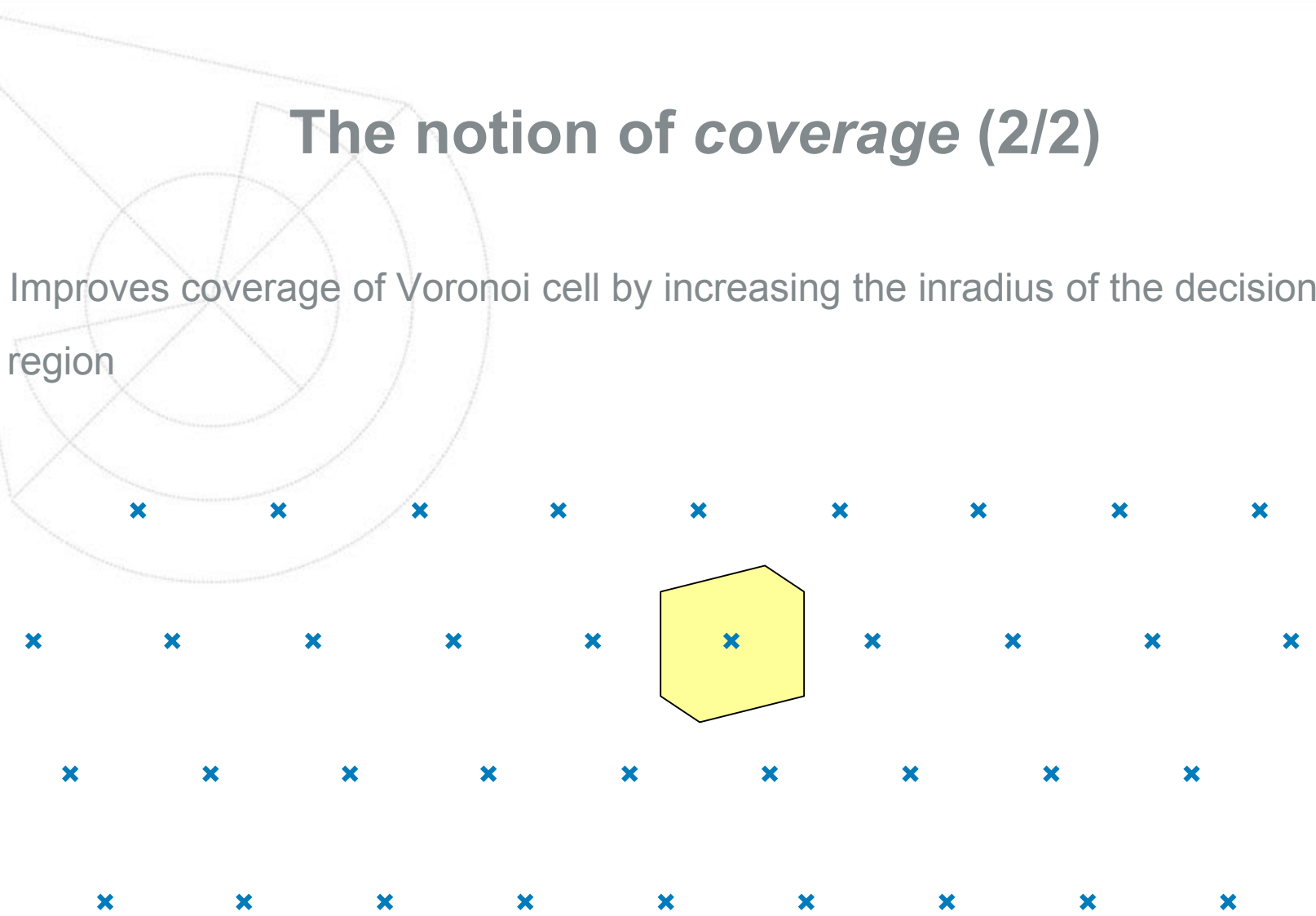


(Figure co-authored with Dr. Karen Su, University of Cambridge )



# The notion of coverage (2/2)

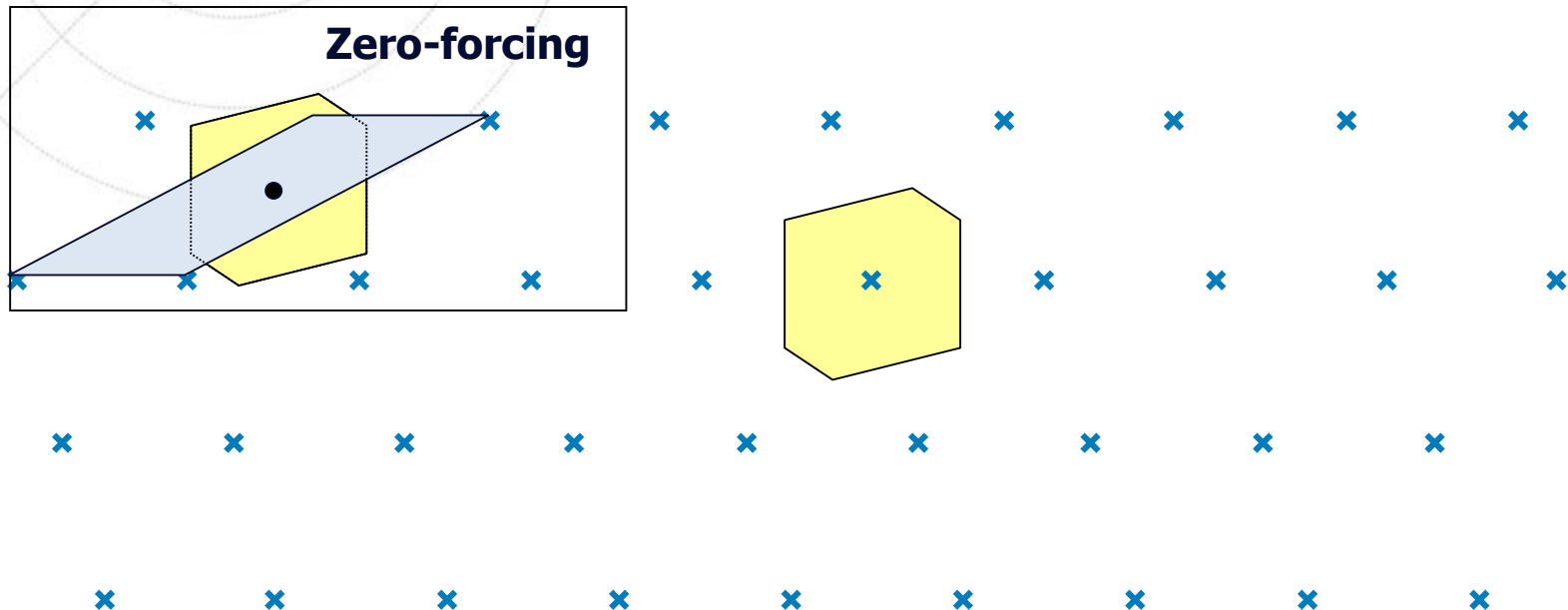
Improves coverage of Voronoi cell by increasing the inradius of the decision region



(Figure co-authored with Dr. Karen Su, University of Cambridge)

# The notion of coverage (2/2)

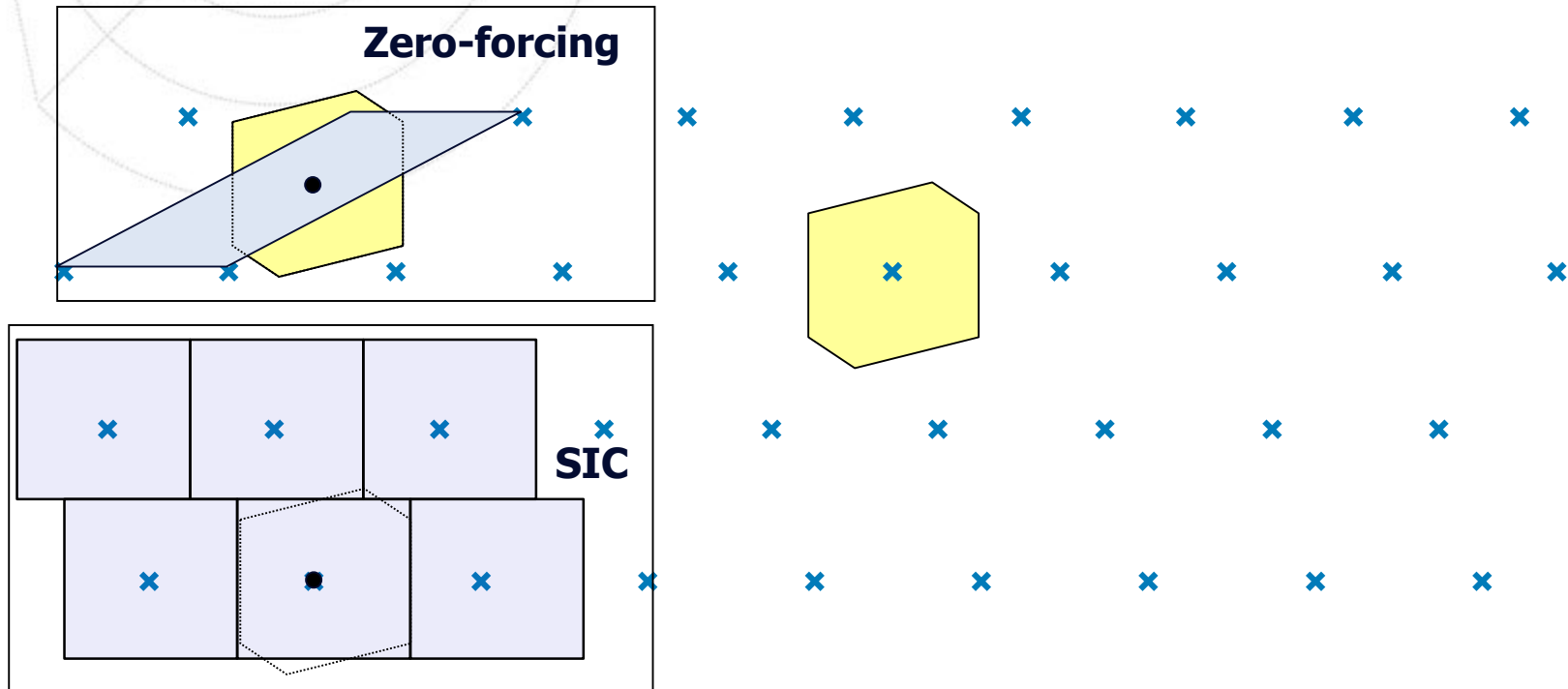
Improves coverage of Voronoi cell by increasing the inradius of the decision region



(Figure co-authored with Dr. Karen Su, University of Cambridge )

# The notion of coverage (2/2)

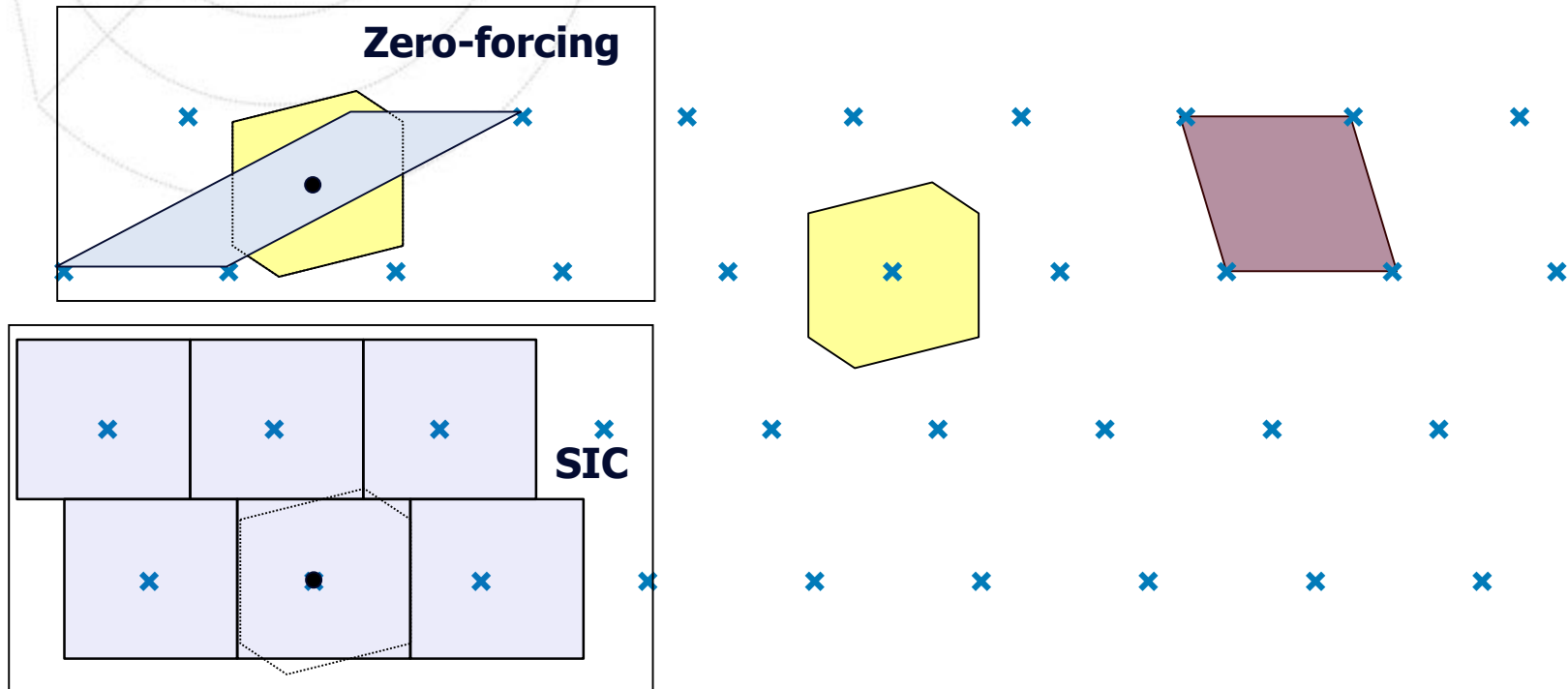
Improves coverage of Voronoi cell by increasing the inradius of the decision region



(Figure co-authored with Dr. Karen Su, University of Cambridge )

# The notion of coverage (2/2)

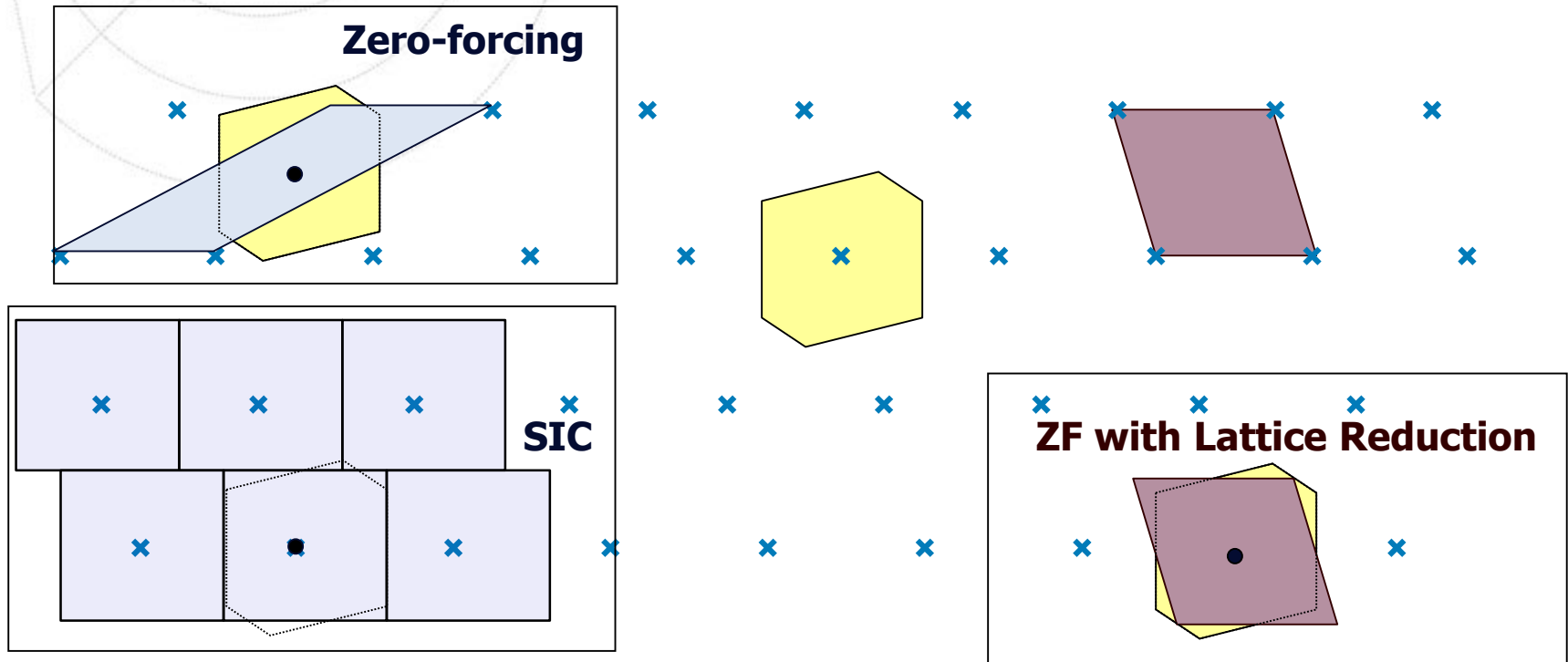
Improves coverage of Voronoi cell by increasing the inradius of the decision region



(Figure co-authored with Dr. Karen Su, University of Cambridge)

# The notion of coverage (2/2)

Improves coverage of Voronoi cell by increasing the inradius of the decision region



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# Two approaches for MIMO

## Space-Time Coding

- Increase diversity (slope of the BER curves).

## Spatial-multiplexing

- Increase spectral efficiency. Preferable to aim at SM [Lozano & Jindal 2010]



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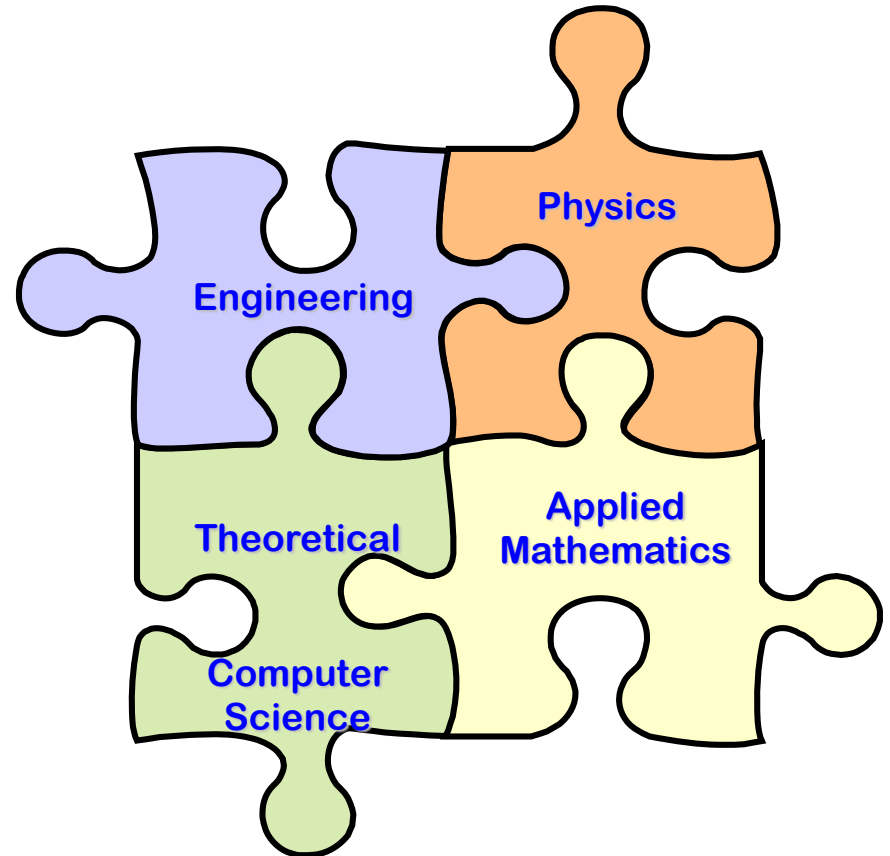
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# MIMO detection brings together...

- *Information Theory*
- *Coding Theory*
- *Detection and Estimation Theory*
- *Statistical signal processing*
- *Algorithms*
- *Optimization*
- *Pattern Recognition*
- *Machine Learning*
- *Cryptography*



# Most used techniques

Linear

- Zero Forcing (ZF)
- Minimum Mean Squared Error (MMSE)

Non-Linear

- V-BLAST (OSIC with ZF criterion)
- V-BLAST (OSIC with MMSE criterion)
- Lattice Reduction Aided (with ZF criterion)
- Lattice Reduction Aided (with MMSE criterion)
- Sphere decoder (with different enumerations)
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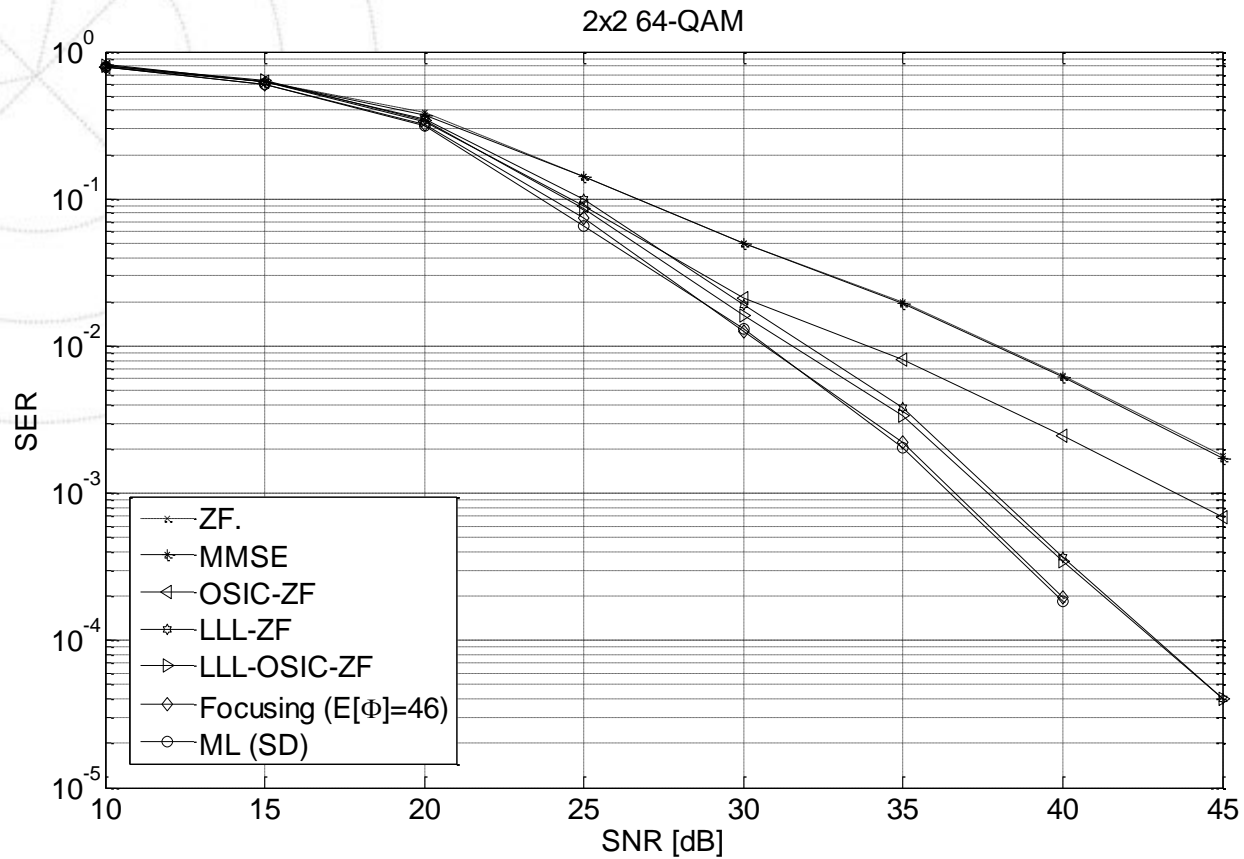
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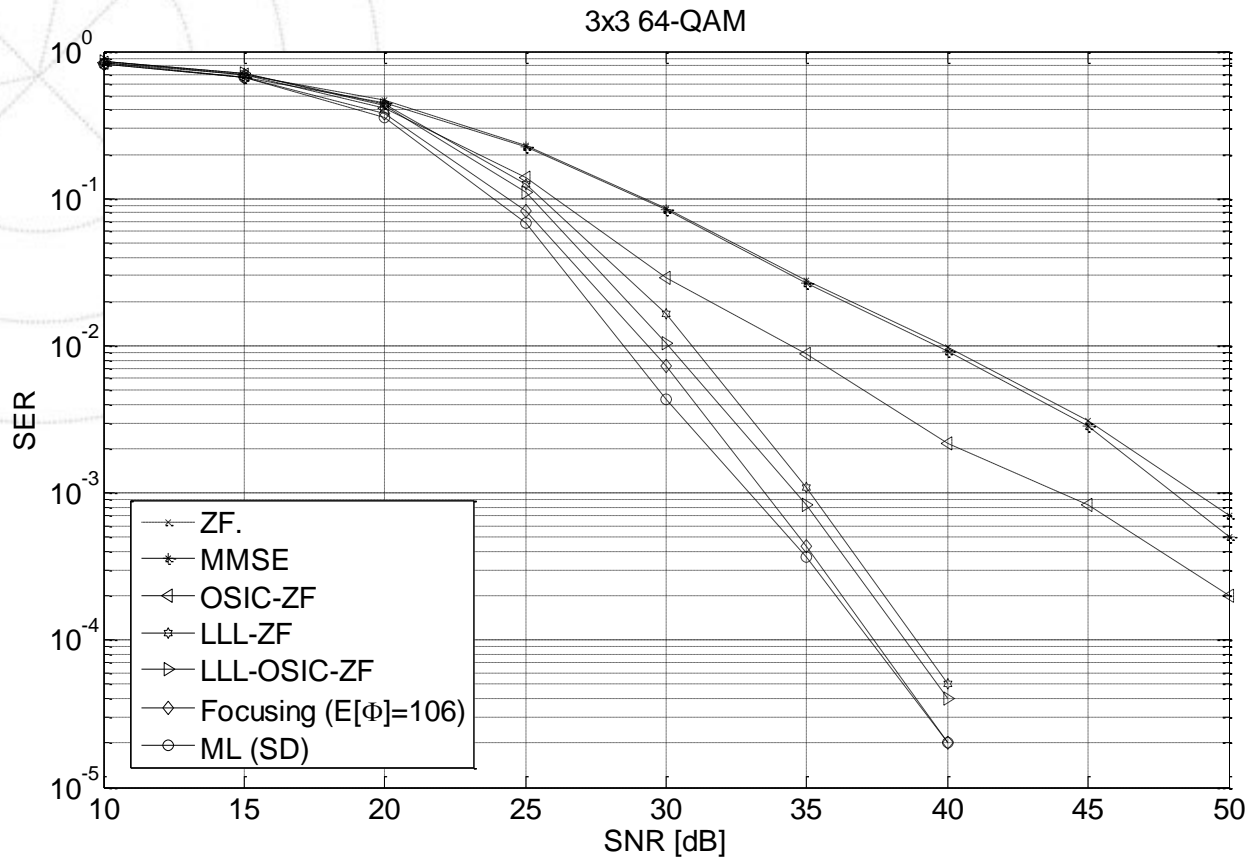
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# Performance

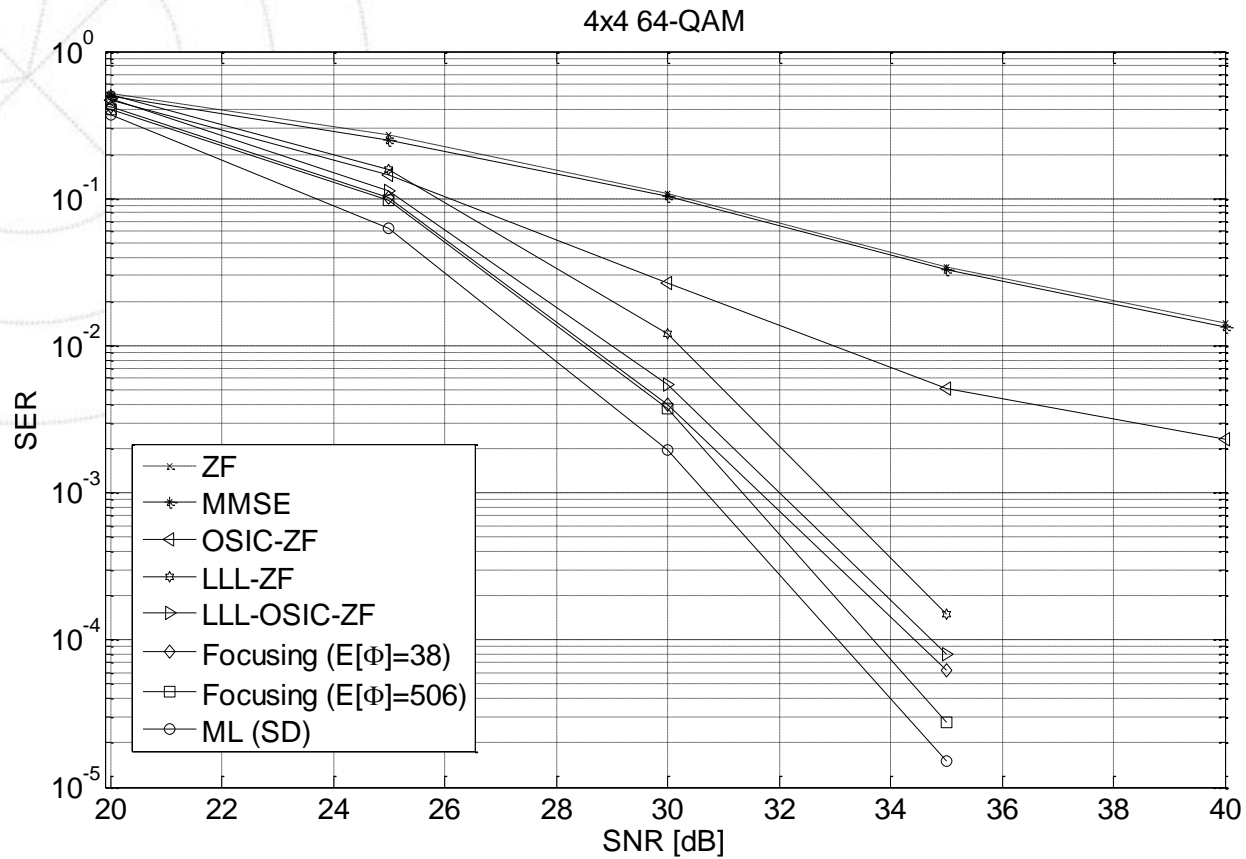


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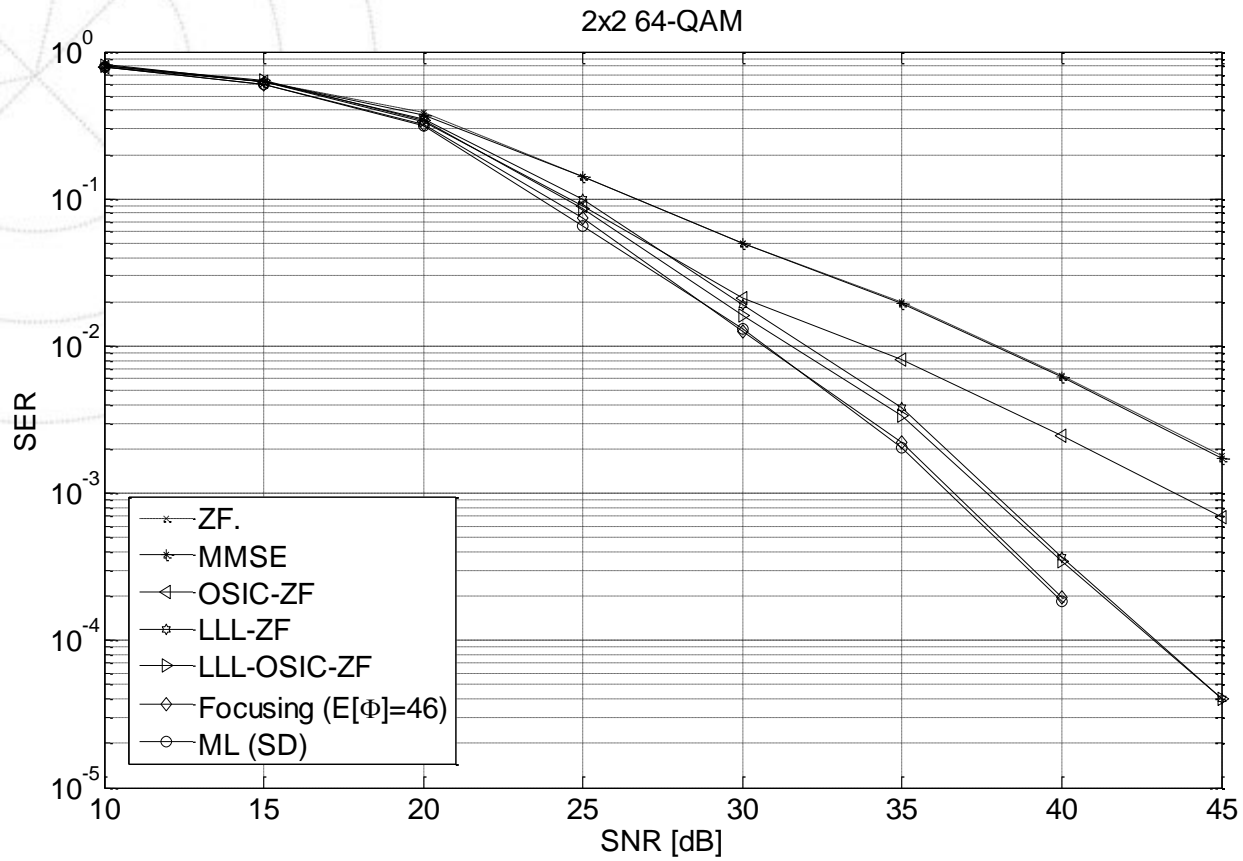




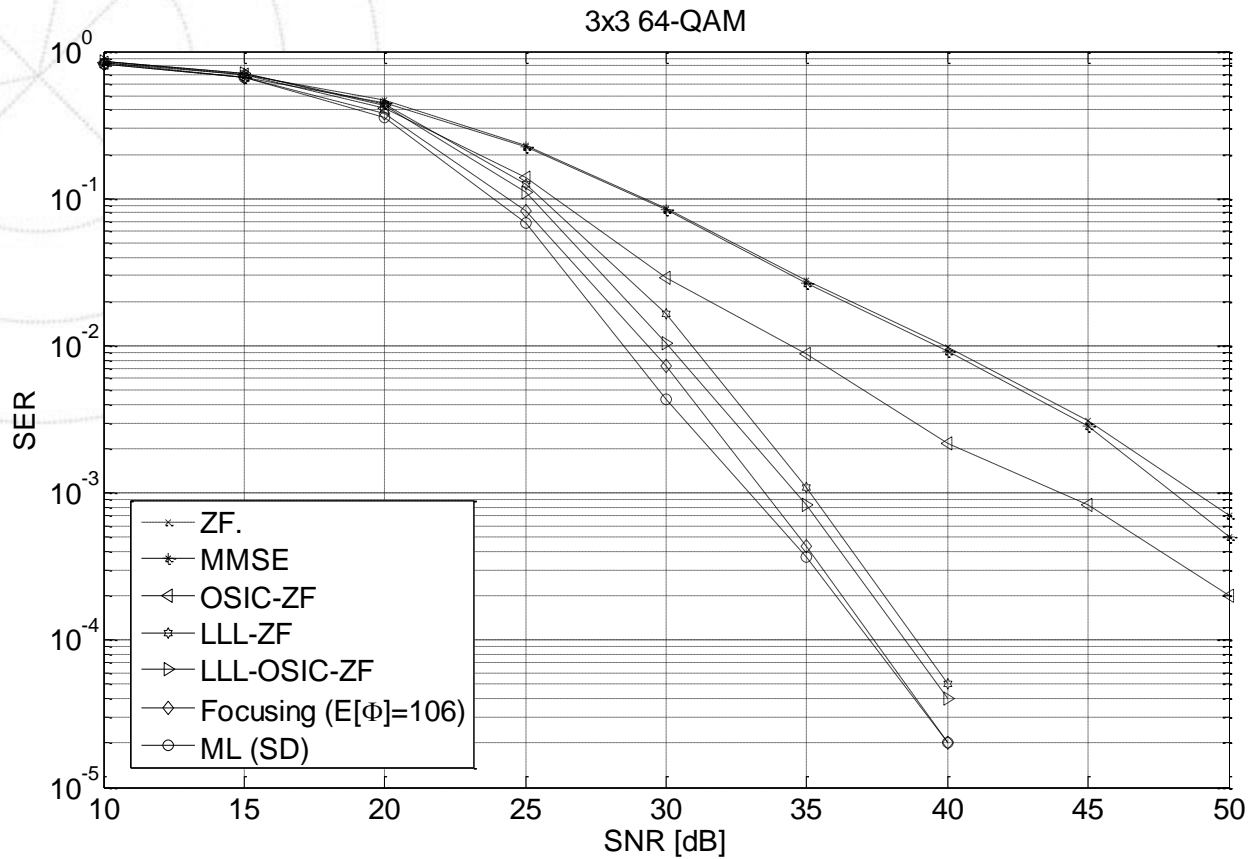
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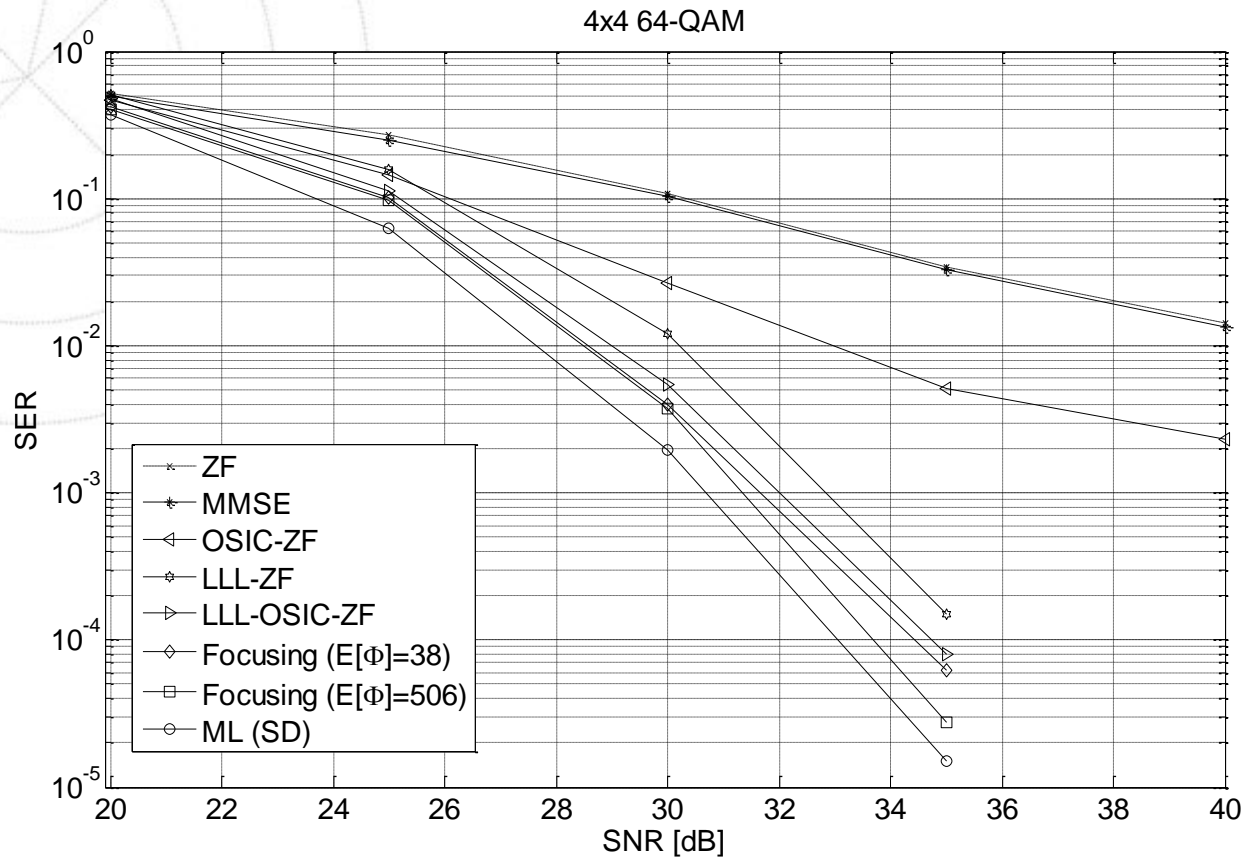
# Performance



# Performance



# Performance



# LTE-Advanced user Equipment categories

3GPP Release	UE Category	Downlink rate	MIMO Layers	Uplink rate
Release 8	Category 1	10.3 Mbit/s	1	5.2 Mbit/s
Release 8	Category 2	51.0 Mbit/s	2	25.5 Mbit/s
Release 8	Category 3	102.0 Mbit/s	2	51.0 Mbit/s
Release 8	Category 4	150.8 Mbit/s	2	51.0 Mbit/s
Release 8	Category 5	299.6 Mbit/s	4	75.4 Mbit/s
Release 10	Category 6	301.5 Mbit/s	2 or 4	51.0 Mbit/s
Release 10	Category 7	301.5 Mbit/s	2 or 4	102.0 Mbit/s
Release 10	Category 8	2998.6 Mbit/s	8	1497.8 Mbit/s



# State of the art in 2015

- LTE-Advanced: 30 b/s/Hz
- Using 8x8 (8 antennas on each side of the link).
- Efficient detection was still an open problem until 2013.  
[e.g., IEEE Comms Mag Feb. 2012]
- “Randomised SIC”. MCMC: Gibbs sampling is a surprisingly near-optimal solution. Perhaps a revolution.



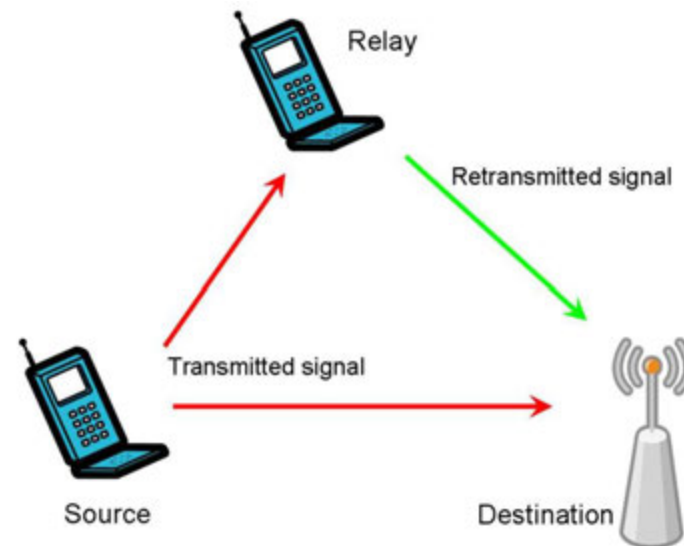
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(In conventional symmetric MIMO)

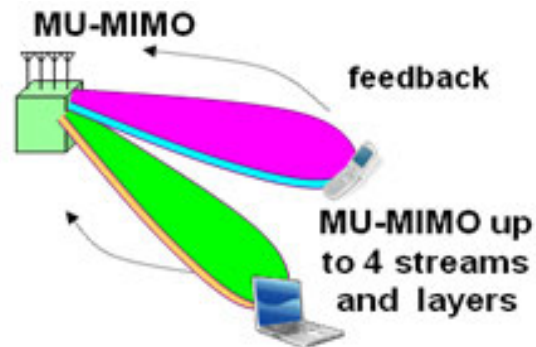
# Under development

- Co-operative relay networks  
(Started with Laneman & Wornel, MIT 2003)



# Broadcast channel Precoding (or MU-MIMO in the LTE jargon) is a dual concept to spatial multiplexing

- Is the reverse (*dual*) of Spatial Multiplexing
- Base station transmits to all and each terminal only sees its own signal
- Requires Channel knowledge at the transmitter (of course!)



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# MIMO Processing for 4G and Beyond

Fundamentals  
and Evolution

Edited by  
Mário Marques da Silva  
Francisco A. Monteiro



Includes an introduction to  
*MIMO detection techniques*

*(CRC Press - Taylor and Francis, June 2014)*