Design and Application of Random Network Codes (DARNEC'15)
November 4-6, 2015, Istanbul, Turkey

Resource allocation strategies for network-coded service delivery over LTE/LTE-A Systems

Invited Tutorial

by Ioannis Chatzigeorgiou

EPSRC
Engineering and Physical Sciences Research Council

School of Computing and Communications

Lancaster University

Outcomes of the EPSRC-funded R2D2 project
http://www.lancaster.ac.uk/~chatzige/R2D2/

network coding for
Rapid and Reliable Data Delivery (R2D2)

- 18-month EPSRC research project on network coding techniques (February 2014 – July 2015).

- Design novel mathematical frameworks - To identify key relationships between system and channel parameters, understand network dynamics and optimise network-coded architectures.

- Investigate practical aspects - Ultra-reliable communications, delay-constrained applications, energy-efficient architectures

The R2D2 Team @ LU

Ioannis Chatzigeorgiou
Principal Investigator

Andrea Tassi
Postdoctoral Research Associate
(Feb. 2014 – June 2015)

Amjad Saeed Khan
PhD Candidate
(Oct. 2014 - present)

Andrew Jones
MSc by Research
(Sep. 2013 – Aug. 2014)

Tutorial Outline

1) Description and performance evaluation
   - Random Linear Network Coding (RLNC) for broadcasting
   - 'Straightforward' vs. systematic RLNC
   - RLNC for layered services
   - Sparse RLNC
2) Resource allocation for network coded systems
   - Basic concepts of LTE/LTE-A and ultra-reliable layered services
   - Optimizing with respect to the service provider or the users
   - The use of sparse RLNC in LTE/LTE-A systems
3) Concluding remarks

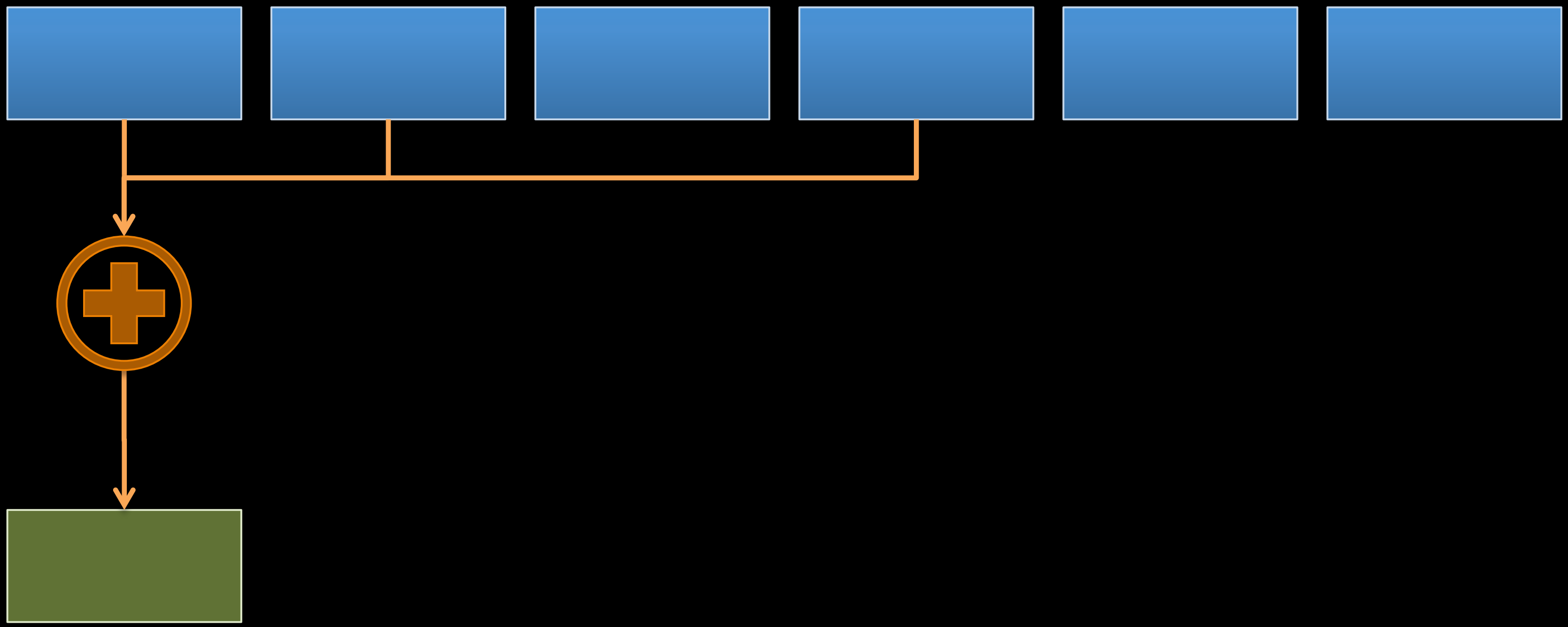
1. Description and performance evaluation

Random Linear (Network) Coding

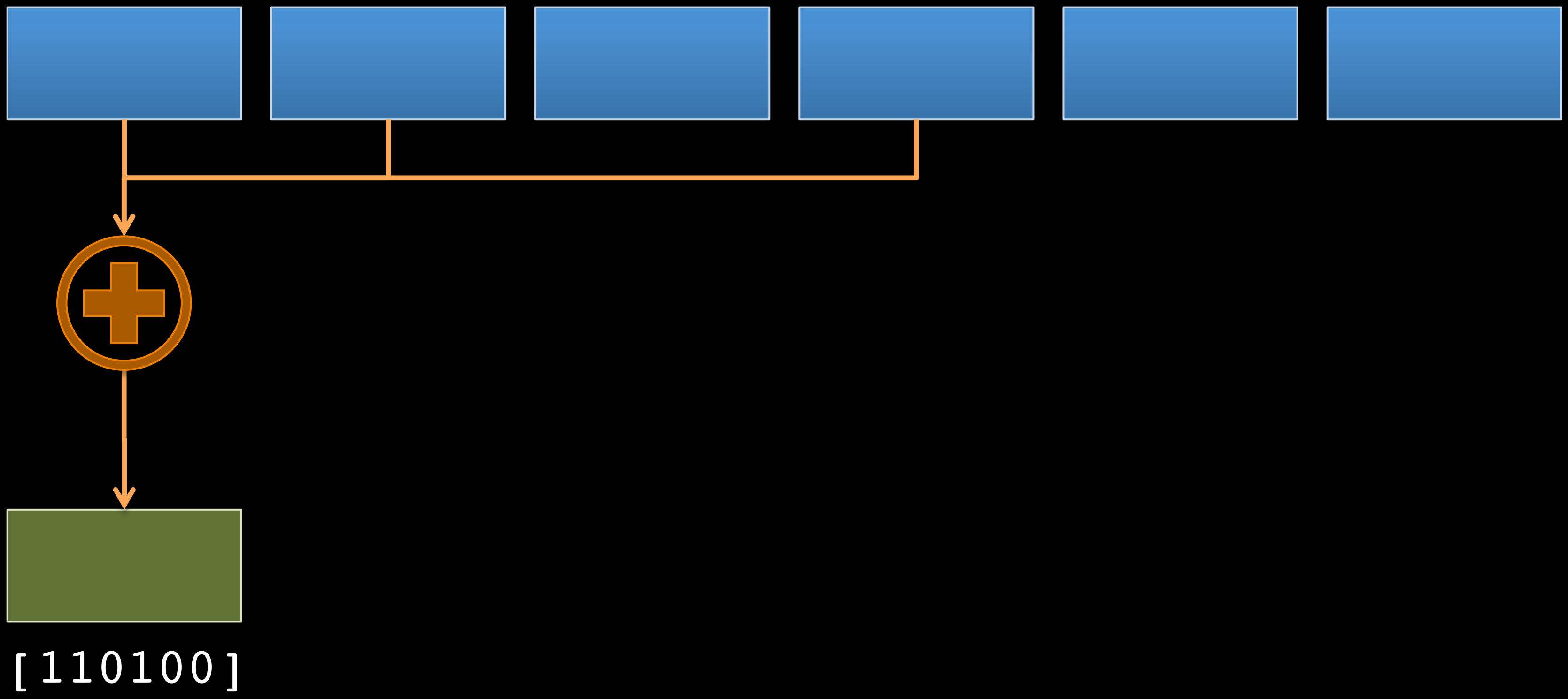
Chunk of information

Random Linear (Network) Coding

Random Linear (Network) Coding



Random Linear (Network) Coding



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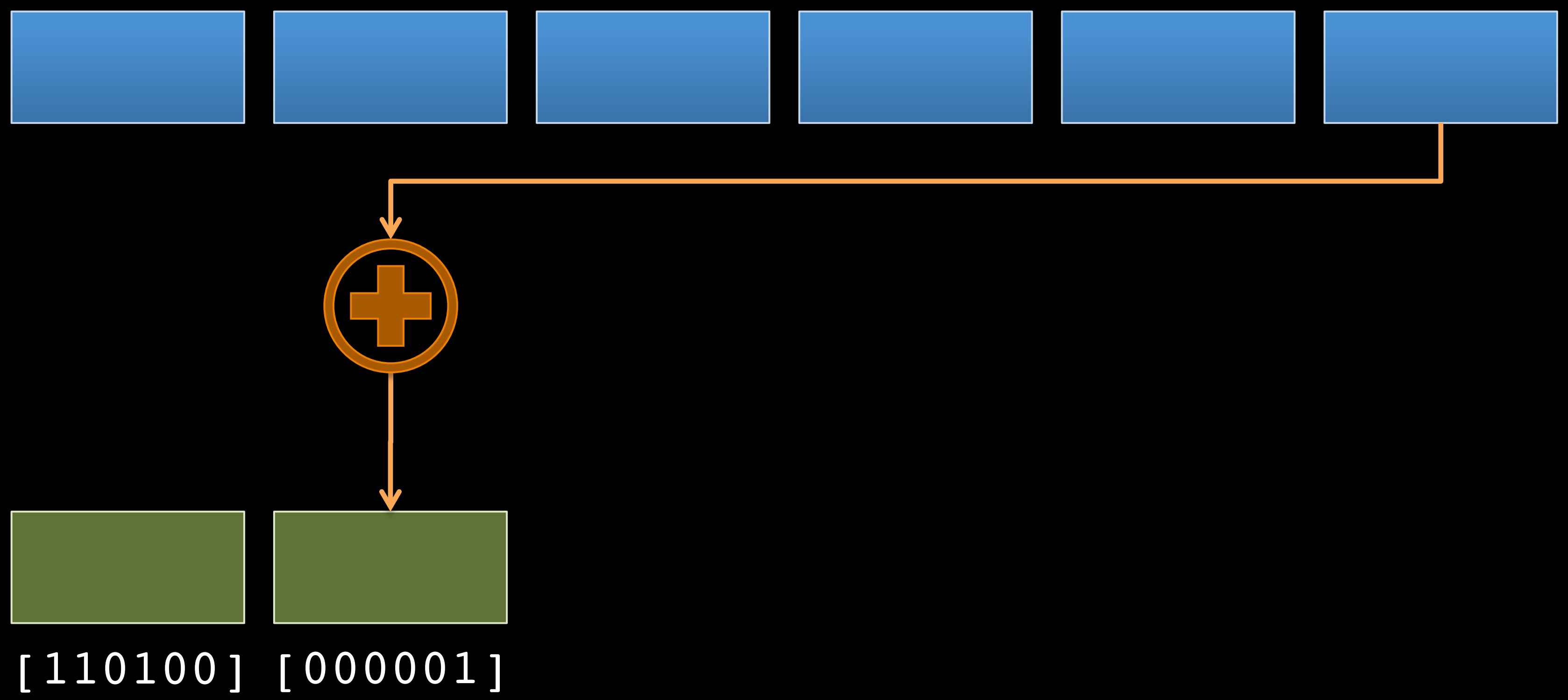
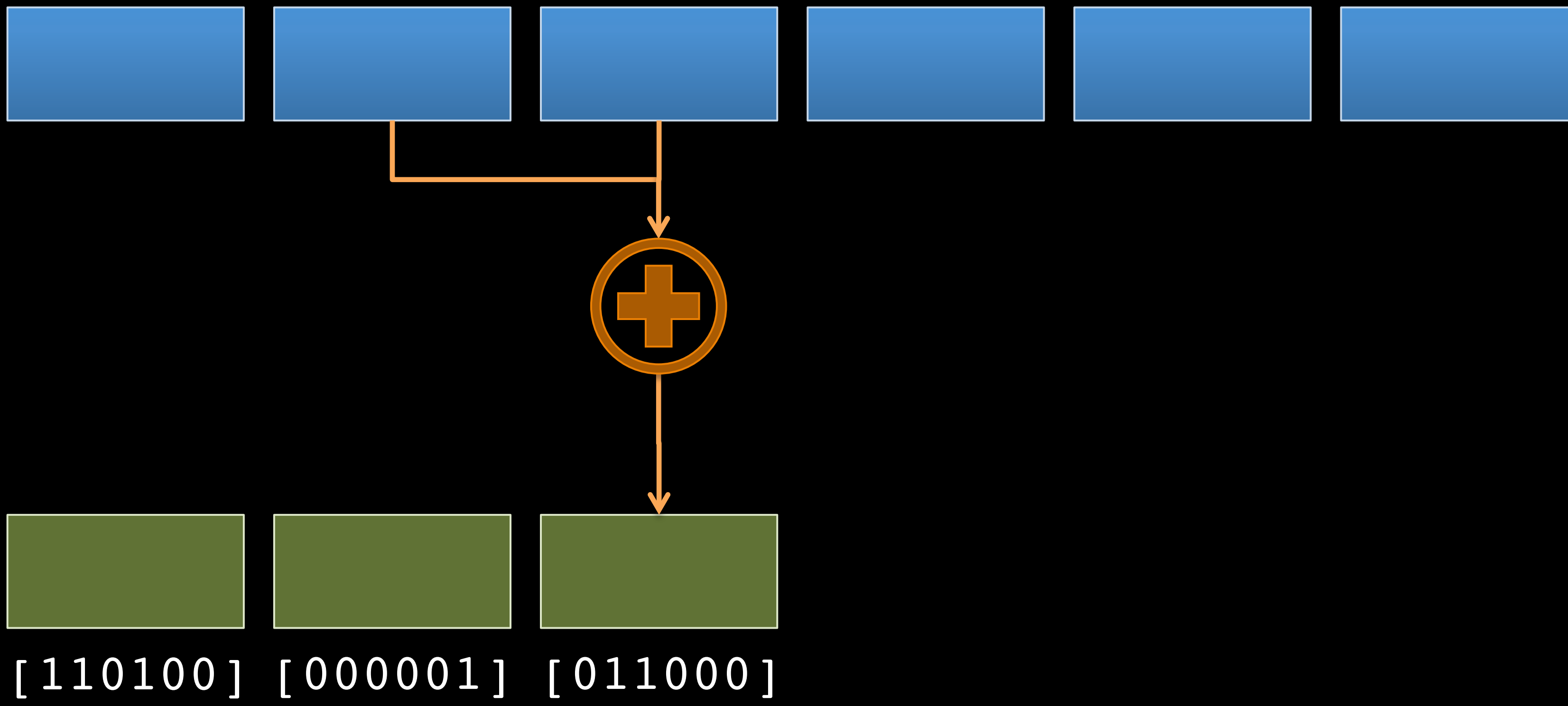
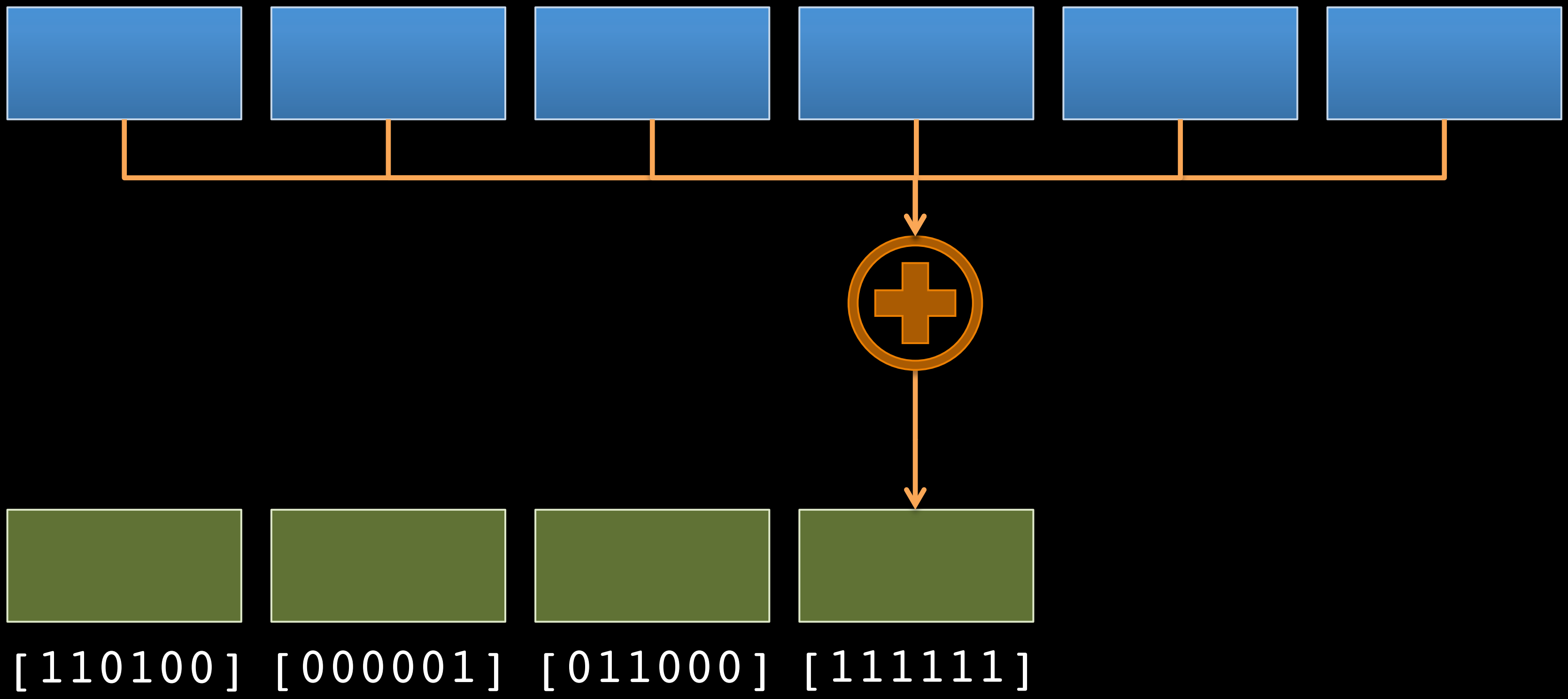


Random Linear (Network) Coding



Random Linear (Network) Coding



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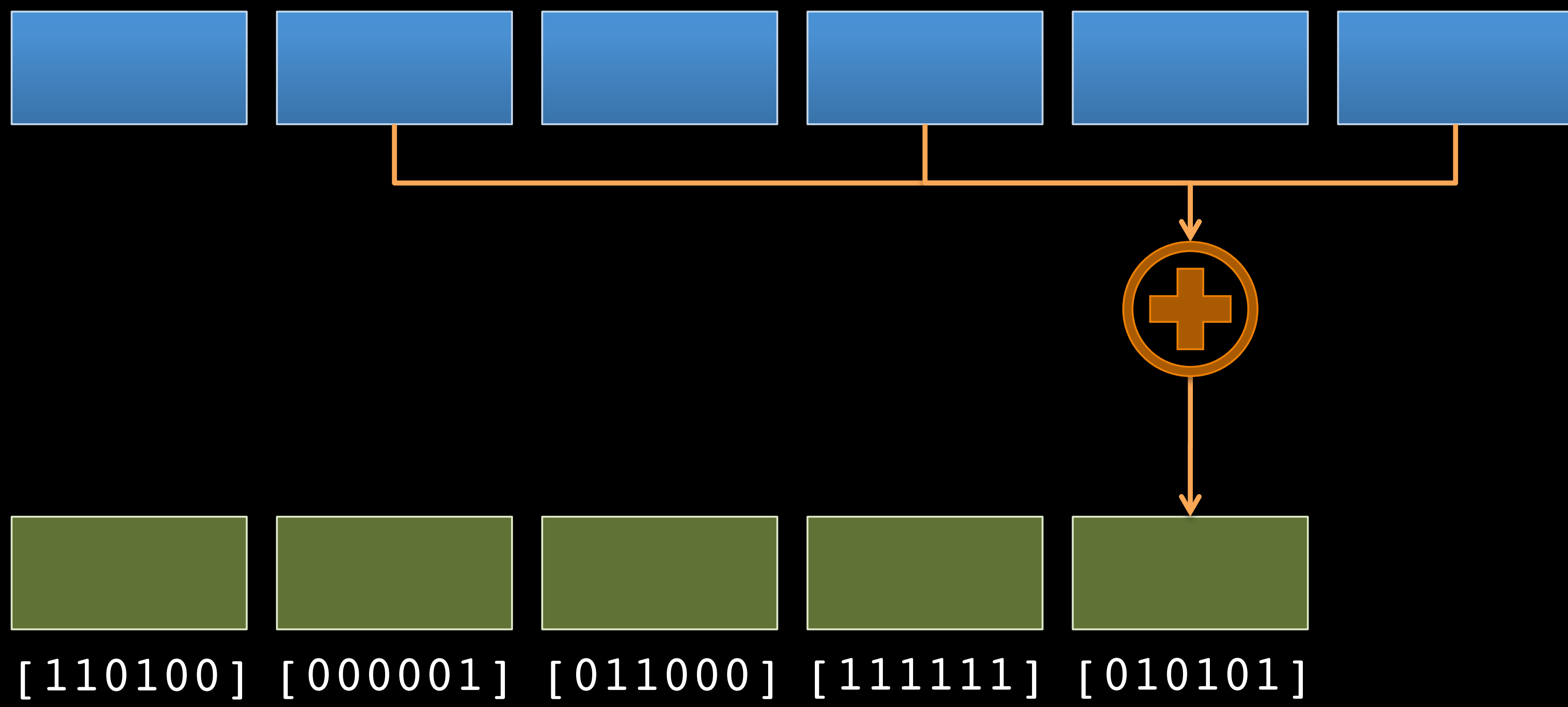

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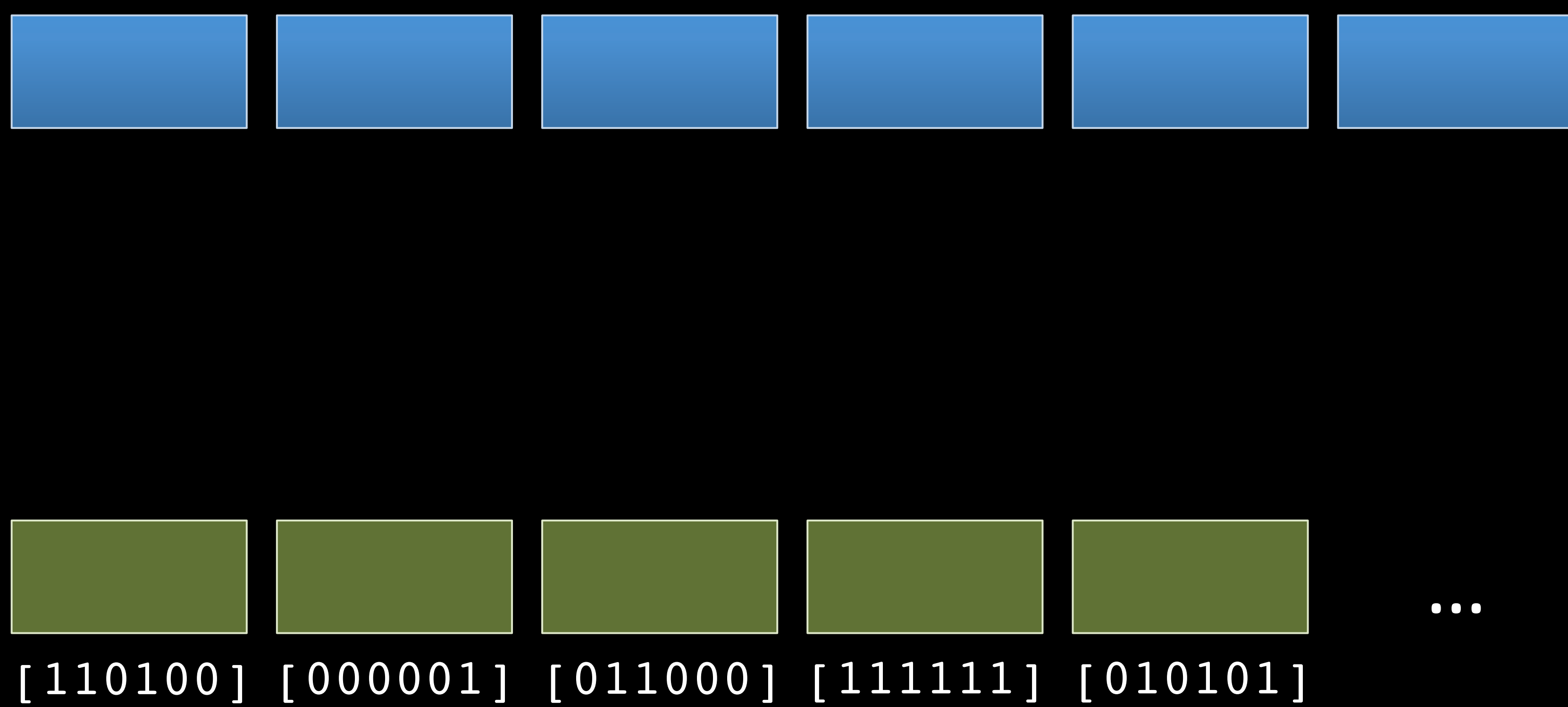

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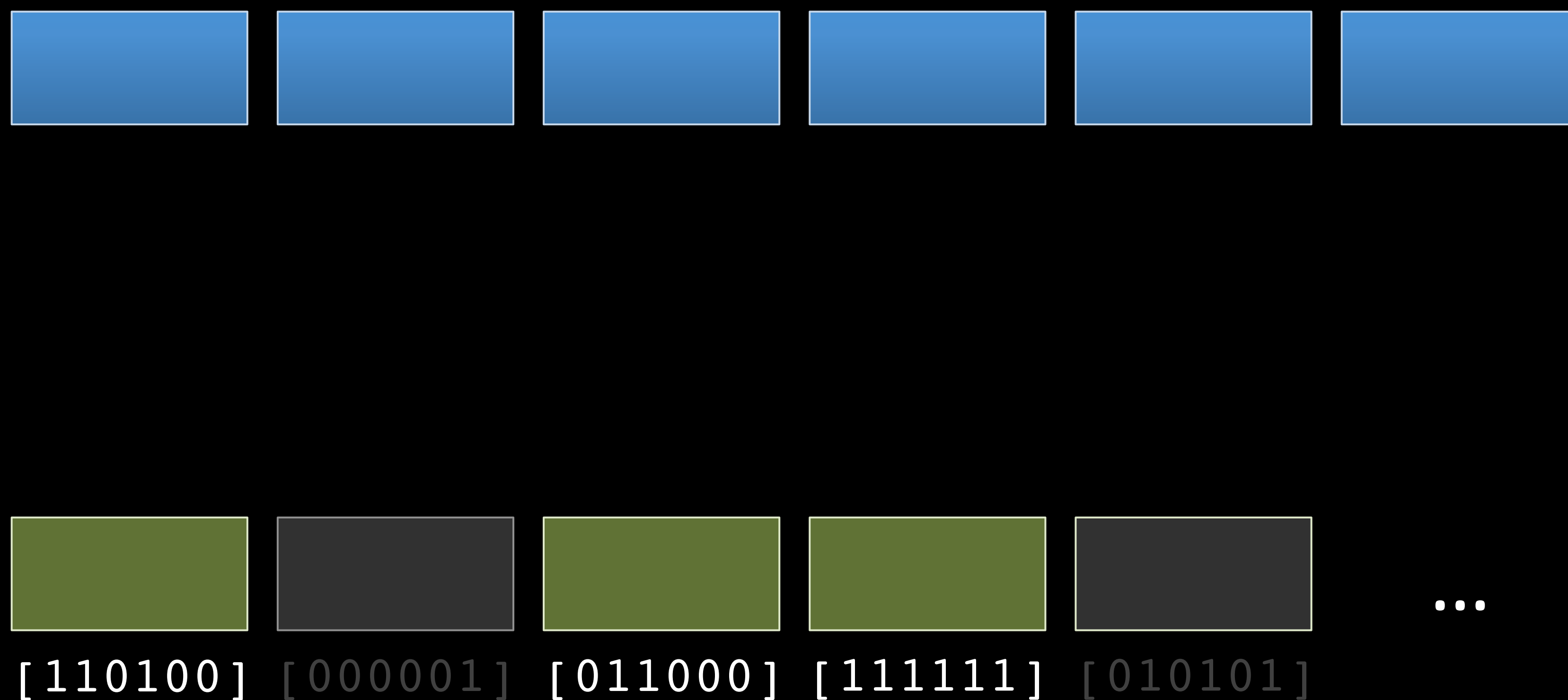
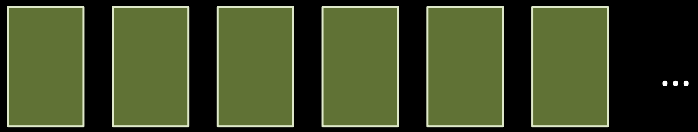


Straightforward and Systematic NC

Straightforward Network Coding

Source packets

Received coded packets



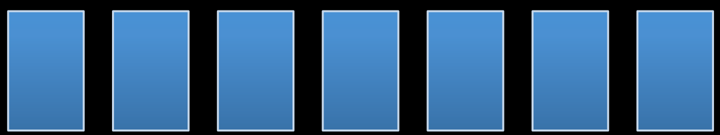
Decoding matrix (example)

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

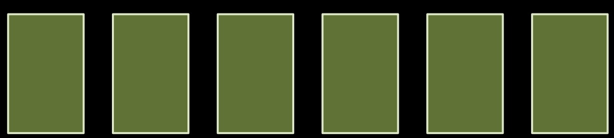
Straightforward and Systematic NC

Straightforward Network Coding

Source packets



Received coded packets



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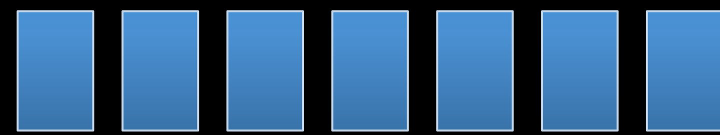
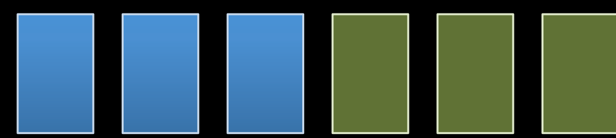
Systematic Network Coding

Source packets



Received systematic & coded packets



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Decoding probability of straightforward NC

- The probability of a receiver decoding all of the K source packets, given that $r \geq K$ coded packets have been successfully received, is the probability of the decoding matrix $\mathbf{G}$ being full rank.

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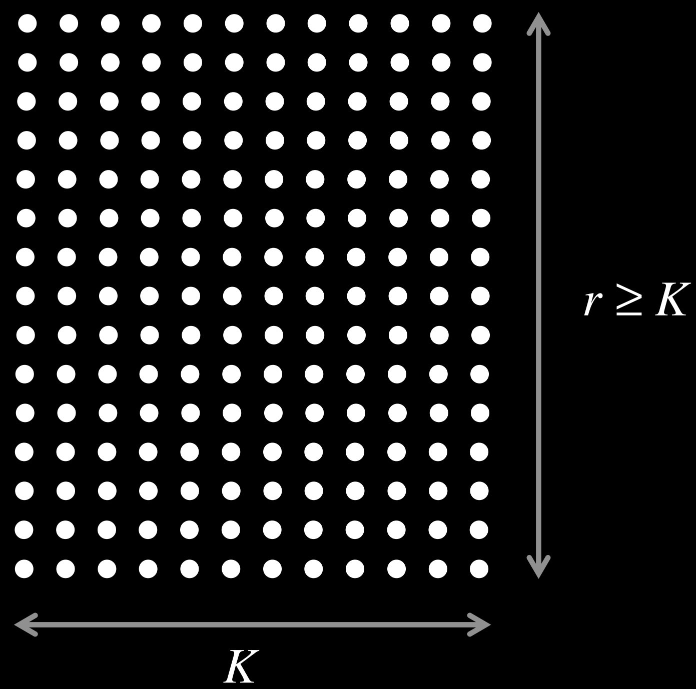

Decoding probability of straightforward NC

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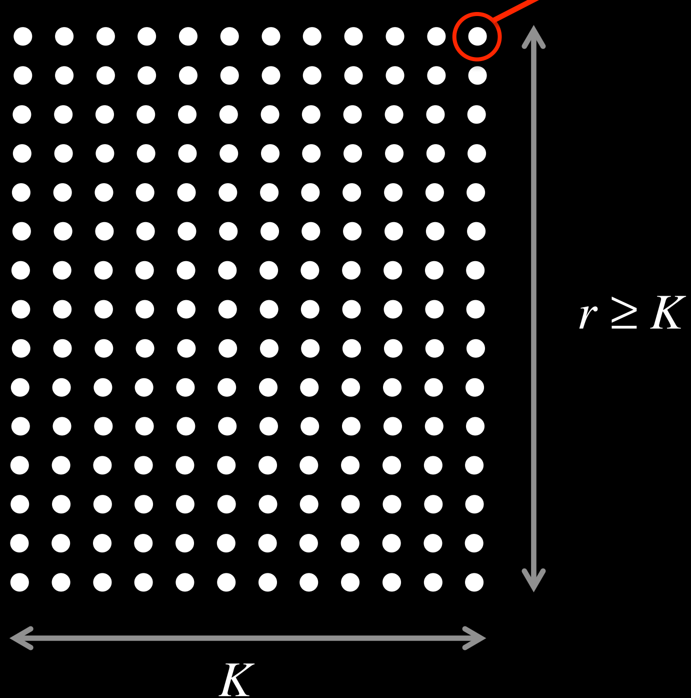
Decoding matrix:

Random element from GF(q), i.e., element • is in set {0, 1, …, q-1} and Pr{•} = 1/q



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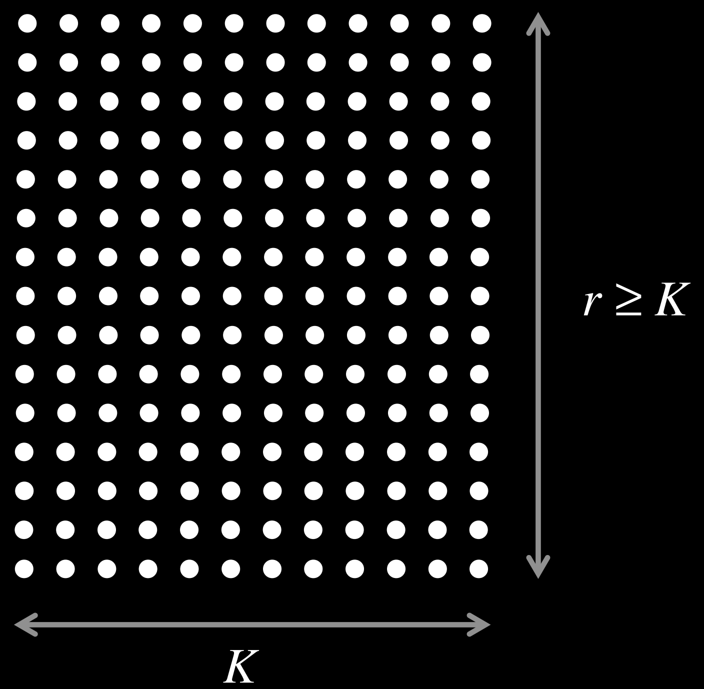

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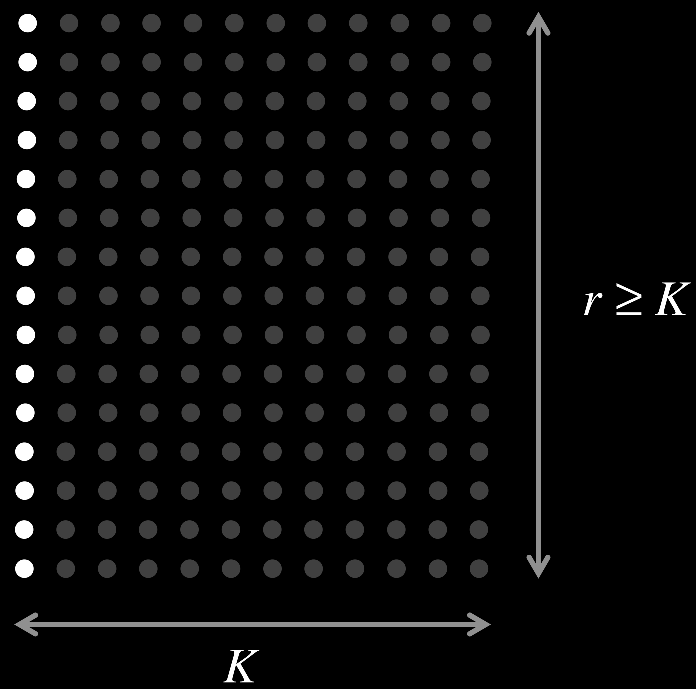

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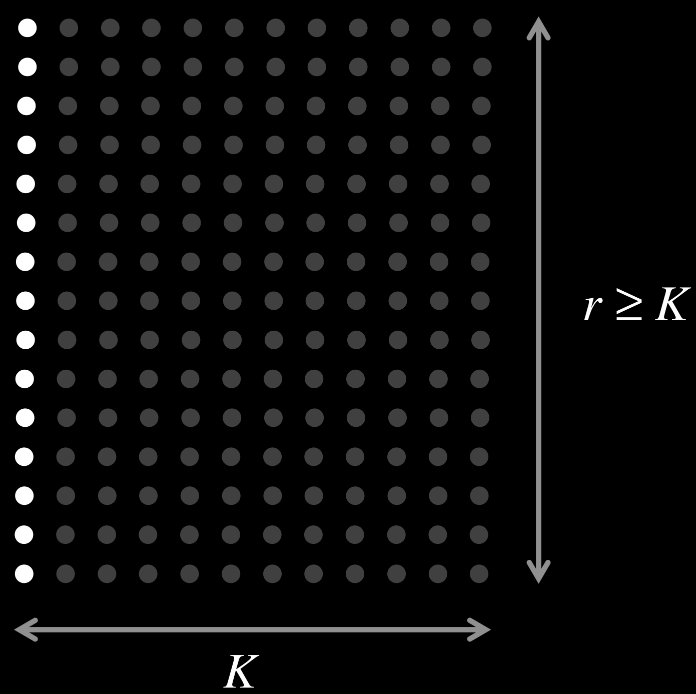

Lin. Ind. Vector in col. 1:

Decoding probability of straightforward NC

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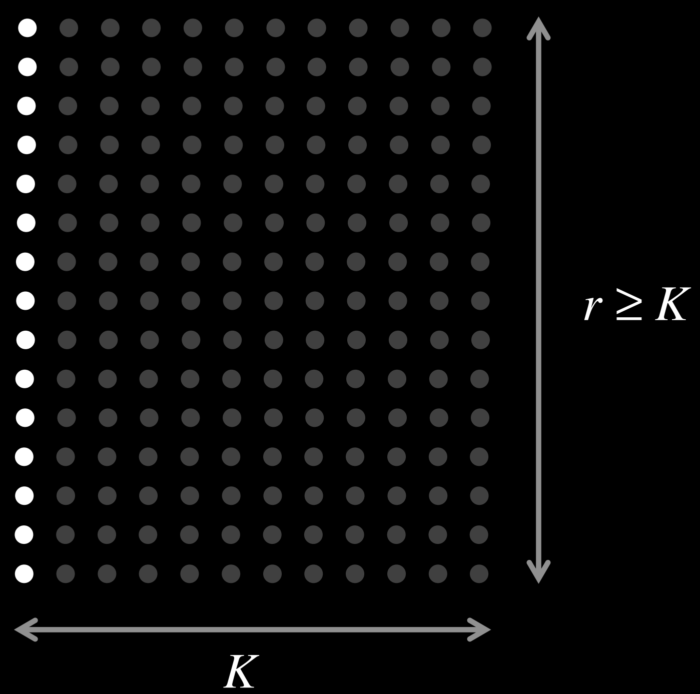

Lin. Ind. Vector in col. 1: $\left(q^{r}-1\right)$

Ignore the all-zero vector and choose col. 1 from the remaining vectors of the r-dimensional space.

Decoding probability of straightforward NC

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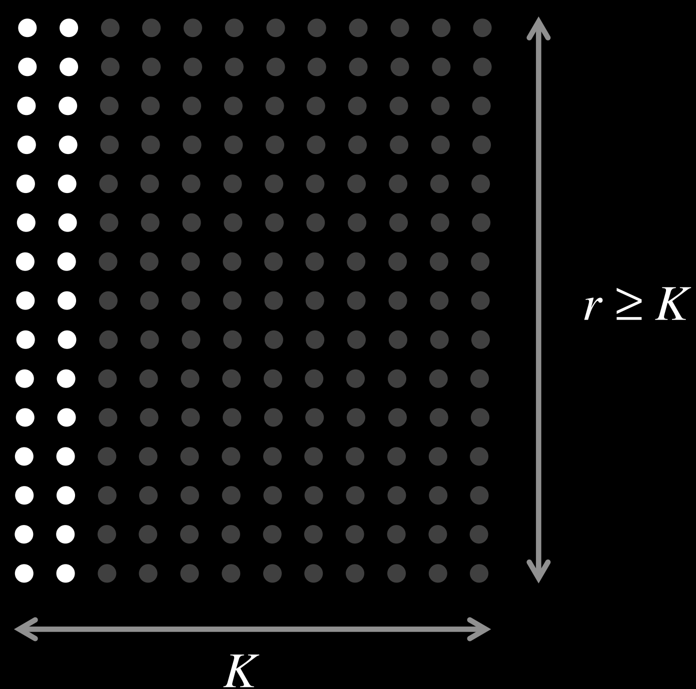
Decoding matrix:



Lin. Ind. Vector in col. 1: $\left(q^r - 1\right)$

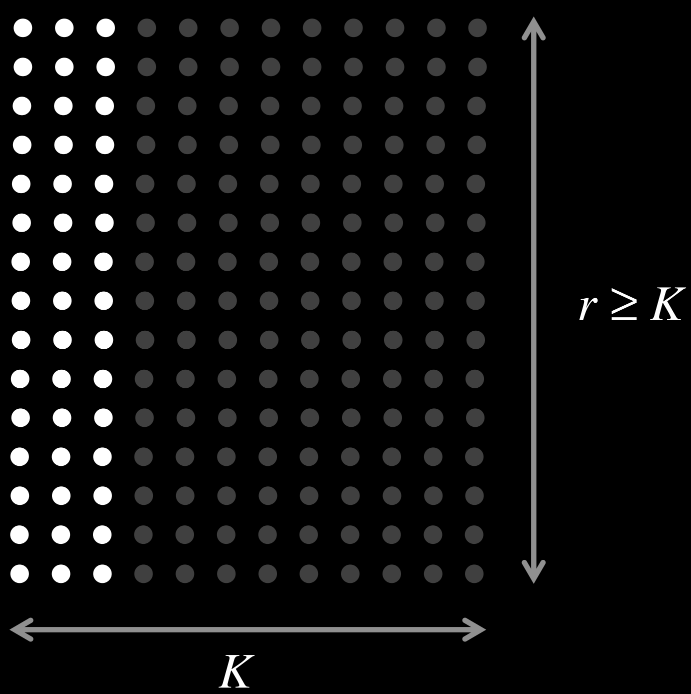
Lin. Ind. Vector in col. 2: $\left(q^r - q\right)$

Ignore the 1-dimensional subspace generated by col. 1 and choose col. 2 from the remaining vectors of the r-dimensional space.

Decoding probability of straightforward NC

- The probability of a receiver decoding all of the K source packets, given that $r \geq K$ coded packets have been successfully received, is the probability of the decoding matrix $\mathbf{G}$ being full rank.

Decoding matrix:



Lin. Ind. Vector in col. 1: $\left(q^r - 1\right)$

Lin. Ind. Vector in col. 2: $\left(q^r - q\right)$

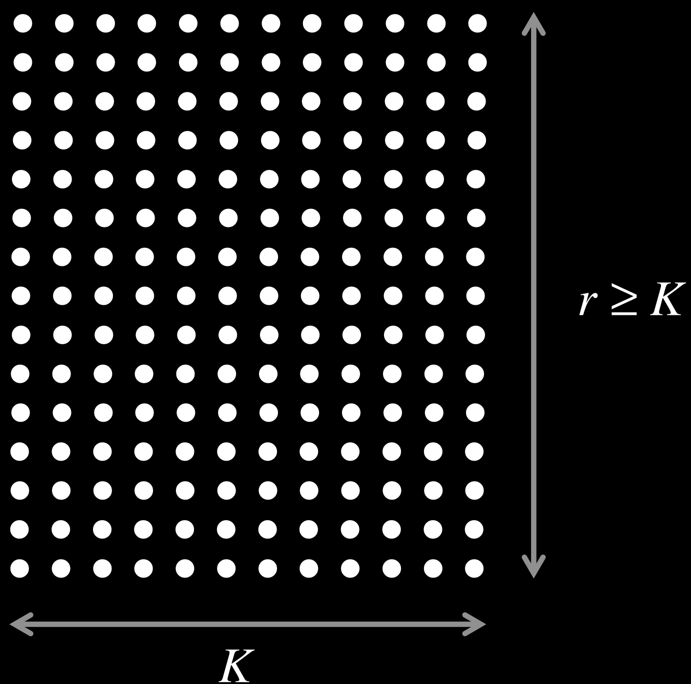
Lin. Ind. Vector in col. 3: $\left(q^r - q^2\right)$

Ignore the 2-dimensional subspace generated by col. 1 and col. 2 and choose col. 3 from the remaining vectors of the r-dimensional space.

Decoding probability of straightforward NC

- The probability of a receiver decoding all of the K source packets, given that $r \geq K$ coded packets have been successfully received, is the probability of the decoding matrix $\mathbf{G}$ being full rank.

Decoding matrix:



Lin. Ind. Vector in col. 1: $\left(q^r - 1\right)$

Lin. Ind. Vector in col. 2: $\left(q^r - q\right)$

Lin. Ind. Vector in col. 3: $\left(q^r - q^2\right)$

.
.
.

Lin. Ind. Vector in col. K: $\left(q^r - q^{K-1}\right)$

Decoding probability of straightforward NC

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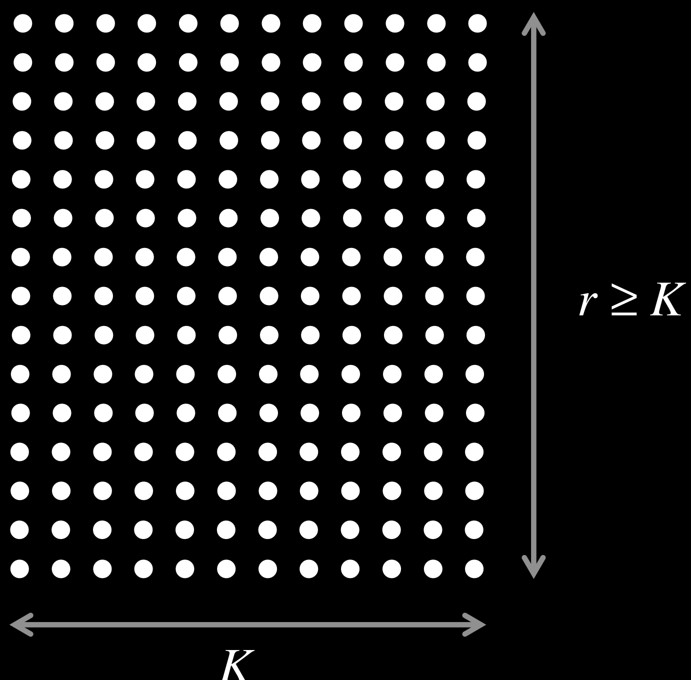
Decoding matrix:



The number of full-rank r x K matrices is:

$$\prod_{i=0}^{K-1}\left(q^r - q^i\right)$$

Decoding probability of straightforward NC

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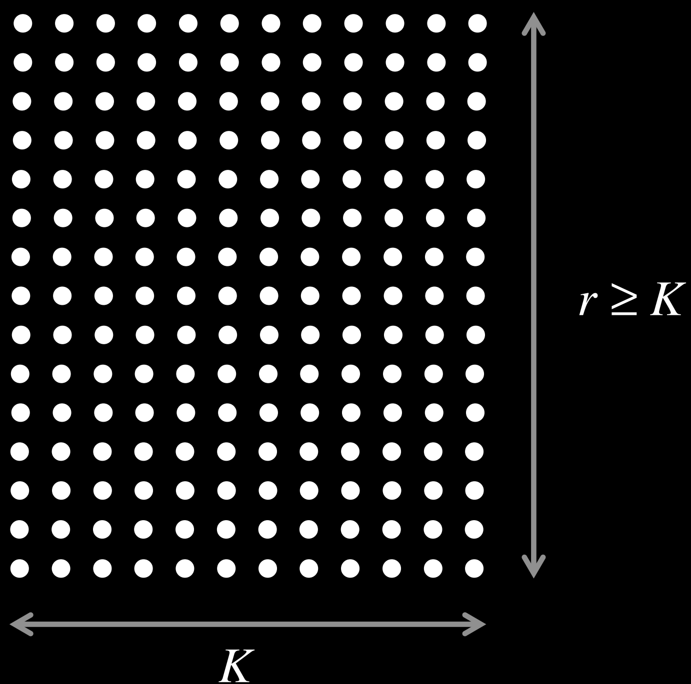

Decoding probability:

$$w_K(r) = \frac{\prod_{i=0}^{K-1}\left(q^r - q^i\right)}{q^{rK}}$$

$$= \prod_{i=0}^{K-1}\left[1 - q^{-(r-i)}\right]$$

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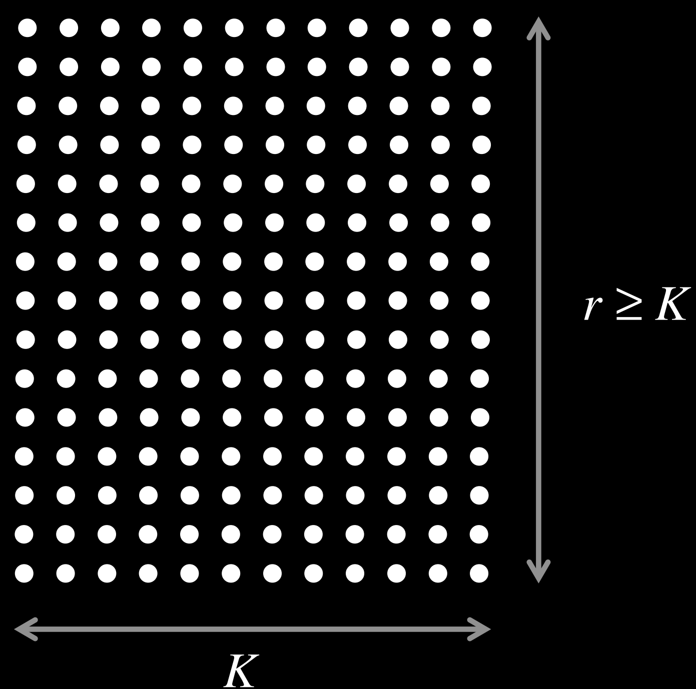

Decoding probability:

$$
\begin{aligned}
w_K(r) &= \frac{\prod_{i=0}^{K-1}\left(q^r - q^i\right)}{q^{rK}} \\
&= \prod_{i=0}^{K-1}\left[1 - q^{-(r-i)}\right] \\
&> 1 - \left[q^{-r} + \ldots + q^{-(r-K+1)}\right] \\
&= 1 - \frac{q^K - 1}{q^r\,(q-1)}
\end{aligned}
$$

Decoding probability of systematic NC

- For $N \geq K$ transmitted packets, the probability of a receiver decoding all of the K source packets, given that $K \leq r \leq N$ packets have been successfully received can be computed as follows:

Decoding probability of systematic NC

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Decoding matrix:

$$\begin{array}{cc}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & \ddots & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 1 & 0 & 0 & \cdots & 0 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \cdots & \bullet \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\bullet & \bullet & \bullet & \bullet & \bullet & \cdots & \bullet
\end{bmatrix} & r \\
K &
\end{array}$$

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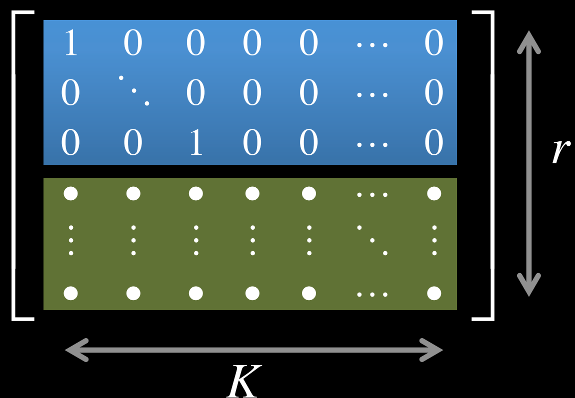

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$$h \qquad K-h$$

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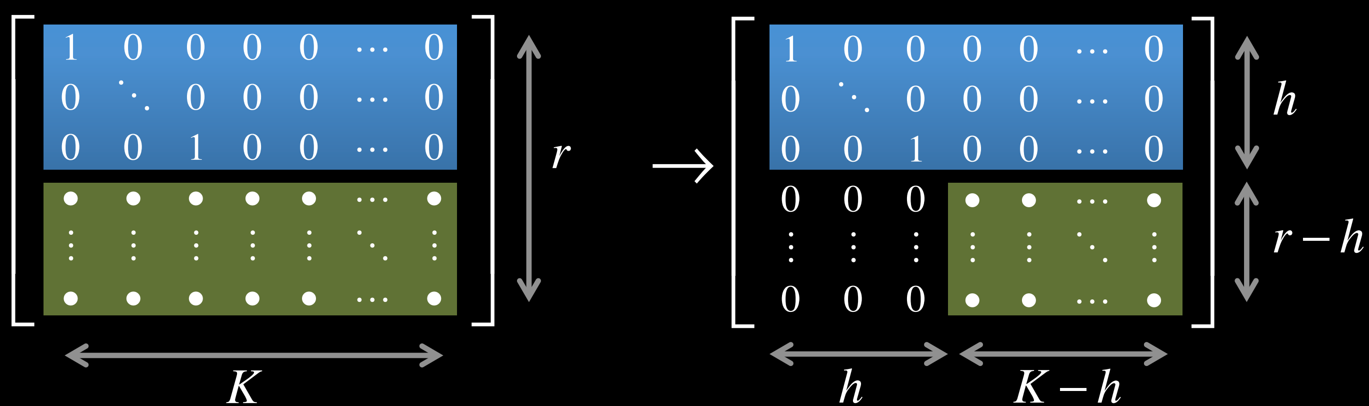

Decoding probability:

$$f_K(r,N) \;=\; \mathrm{P}(h=K) \;+\; \sum_{h=h_{\min}}^{K-1} \mathrm{P}(h<K)\; w_{K-h}(r-h)$$

where:
$h_{\min} = \max(0, r-N+K)$

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Decoding matrix:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \ddots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \cdots & \bullet \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \bullet & \bullet & \bullet & \bullet & \bullet & \cdots & \bullet \end{bmatrix}}_{K} \updownarrow r \quad \rightarrow \quad \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \ddots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \bullet & \bullet & \cdots & \bullet \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \bullet & \bullet & \cdots & \bullet \end{bmatrix} \begin{array}{l} \updownarrow h \\ \updownarrow r-h \end{array} \\ \longleftrightarrow h \quad \longleftrightarrow K-h \end{array}$$

Decoding probability:

$$f_K(r,N) = \mathrm{P}(h=K) + \frac{\sum_{h=h_{\min}}^{K-1} \binom{K}{h} \binom{N-K}{r-h} w_{K-h}(r-h)}{\binom{N}{r}}$$

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$$\qquad\qquad\qquad\qquad h \qquad K-h$$

Decoding probability:

$$f_K(r,N) = \frac{\binom{N-K}{r-K}}{\binom{N}{r}} + \frac{\sum_{h=h_{\min}}^{K-1} \binom{K}{h}\binom{N-K}{r-h} w_{K-h}(r-h)}{\binom{N}{r}}$$

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Systematic vs. Straightforward NC

- Systematic network codes exhibit a higher probability of decoding all of the K packets of a source message than straightforward network codes.

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Proof: We need to show that $f_K(r,N) > w_K(N)$.

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$$\frac{f_K(r,N)}{w_K(N)} = \binom{N}{r}^{-1}\left[\binom{N-K}{r-K}A + \sum_{h=h_{\min}}^{K-1}\binom{K}{h}\binom{N-K}{r-h}B_h\right]$$

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$$> \binom{N}{r}^{-1}\sum_{h=h_{\min}}^{K}\binom{K}{h}\binom{N-K}{r-h}$$

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$$> \binom{N}{r}^{-1} \underbrace{\sum_{h=h_{\min}}^{K} \binom{K}{h}\binom{N-K}{r-h}}_{\binom{N}{r}} \xrightarrow{\text{invoke Vandermonde identities}}$$

$$= 1$$

Average decoding probability

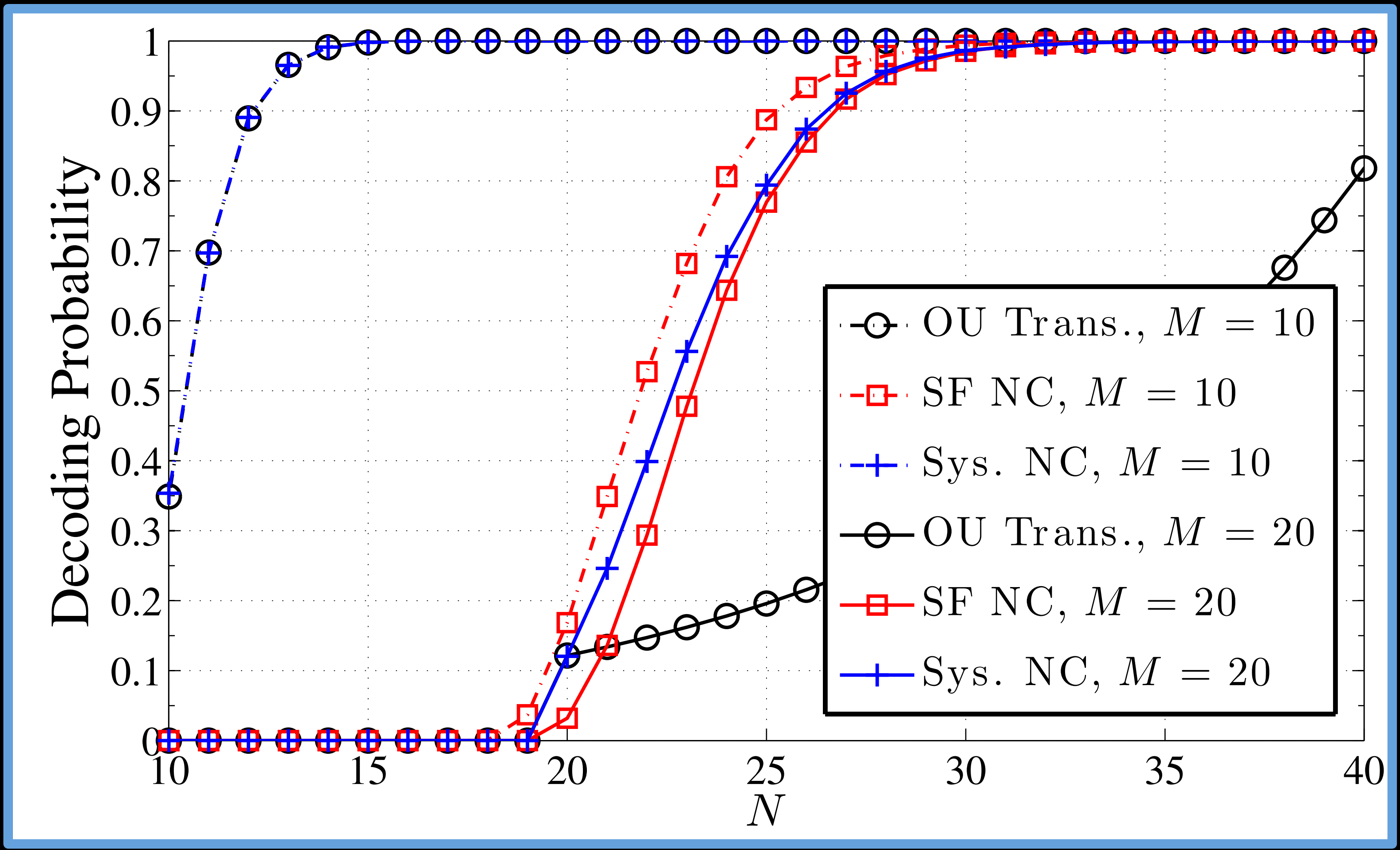
- The probability of a receiver decoding all of the K source packets, after the transmission of $N \geq K$ packets over a channel characterized by a packet erasure probability p, is

SF NC:
$$\mathrm{P}_K(N) = \sum_{r=K}^{N} \binom{N}{r} (1-p)^r \, p^{N-r} w_K(r)$$

Sys. NC:
$$\mathrm{P}_K(N) = \sum_{r=K}^{N} \binom{N}{r} (1-p)^r \, p^{N-r} f_K(r,N)$$

- The probability of recovering at least $M < K$ source packets, when $N \geq M$ packets have been transmitted over a channel with packet erasure probability p, can be *approximated*.

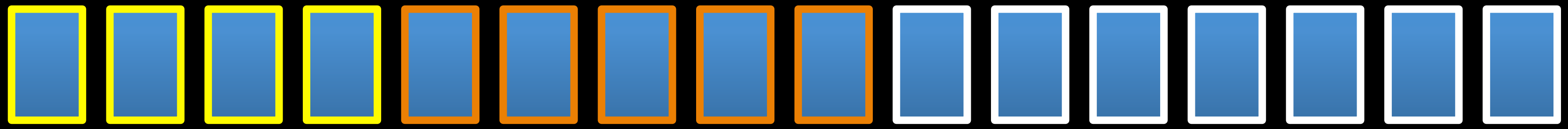
Decoding probability of SF NC and Sys. NC



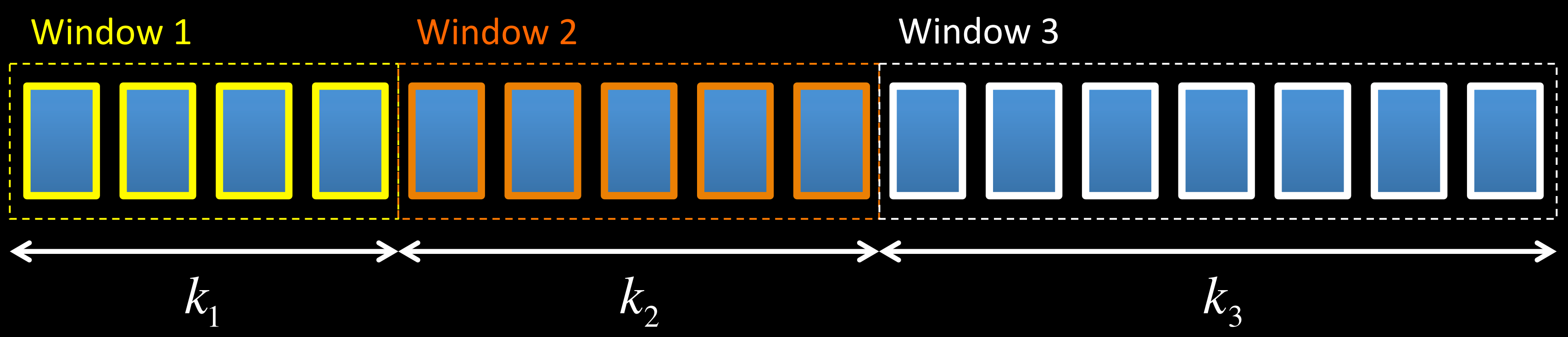
OU: Ordered Un-coded transmission, SF NC: Straightforward Network Coding,

Sys. NC: Systematic Network Coding. (K = 20 and p = 0.1)

Layered NC: Non-Overlapping Window (NOW)



Layered NC: Non-Overlapping Window (NOW)



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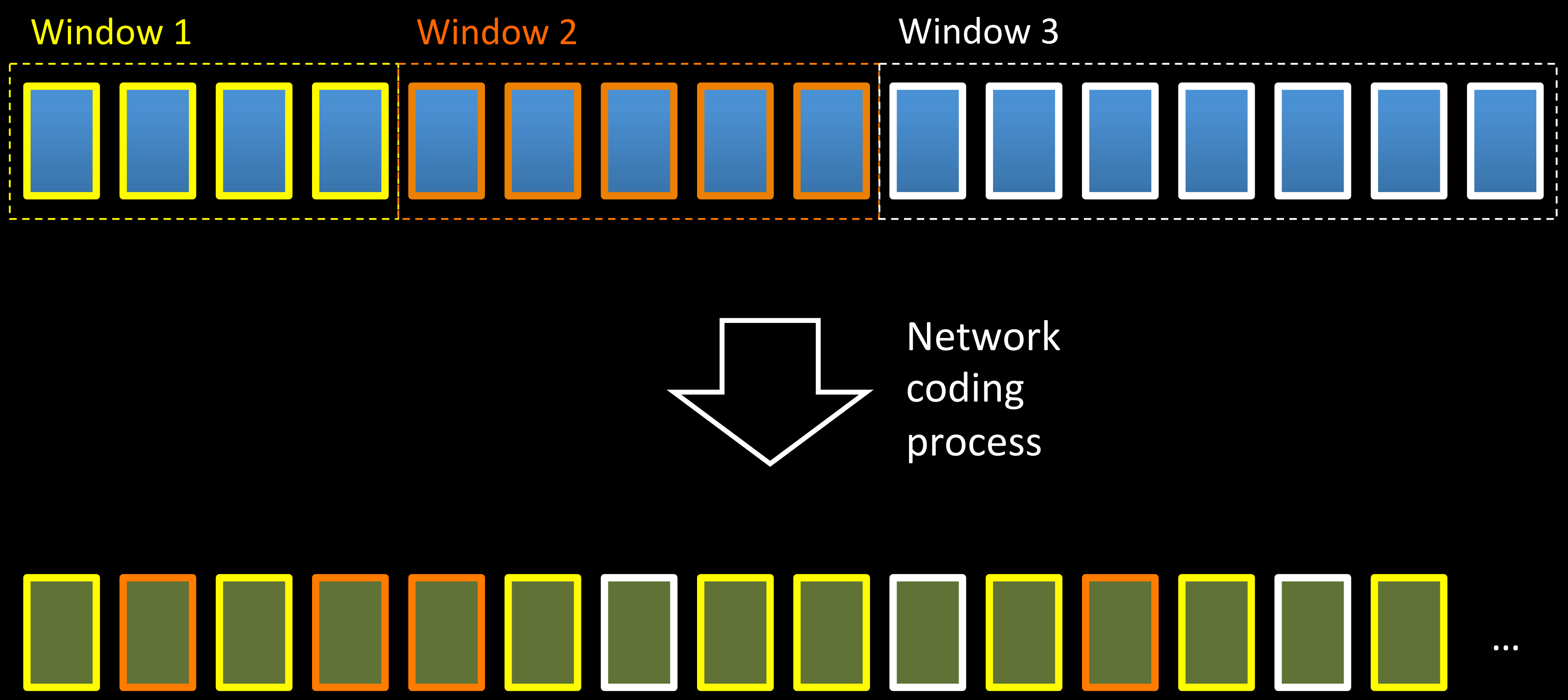

Average decoding probability of NOW-RLNC

- Encoding is performed over each service layer independently of the others.
- The source linearly combines the k_l data packets composing the l-th layer and generates a stream of $n_l \geq k_l$ coded packets.

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- The source linearly combines the k_l data packets composing the l-th layer and generates a stream of $n_l \geq k_l$ coded packets.
- A user can recover the l-th layer if k_l linearly independent coded packets from that layer are collected. The probability of this event is

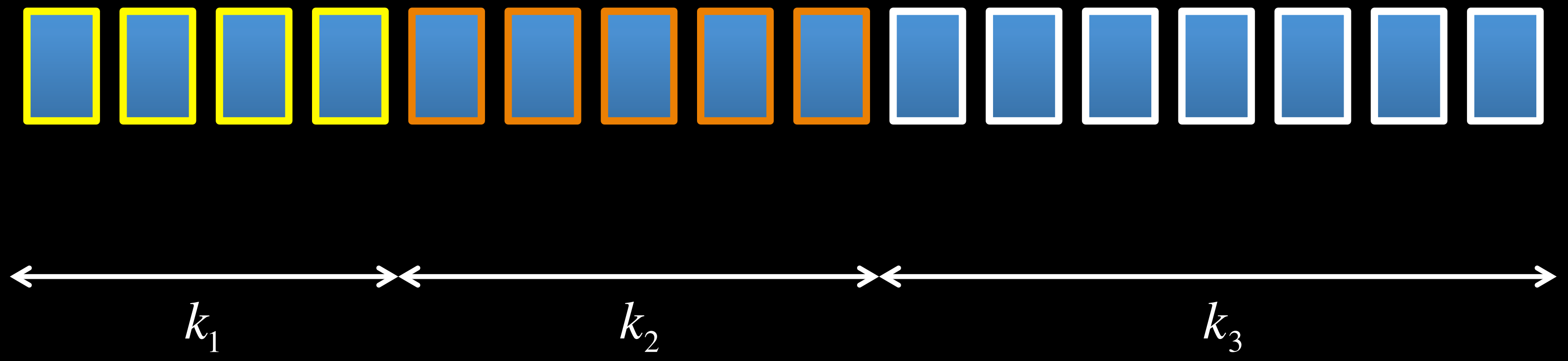
$$P_l(n_l) = \sum_{r=k_l}^{n_l} \binom{n_l}{r} p^{n_l - r} (1-p)^r w_{k_l}(r)$$

where p is the packet erasure probability.

- The probability that the user will recover the first l service layers is

$$D_{\mathrm{NOW}}(n_1, \ldots, n_l) = \prod_{j=1}^{l} P_j(n_j)$$

Layered NC: Expanding Window (EW)



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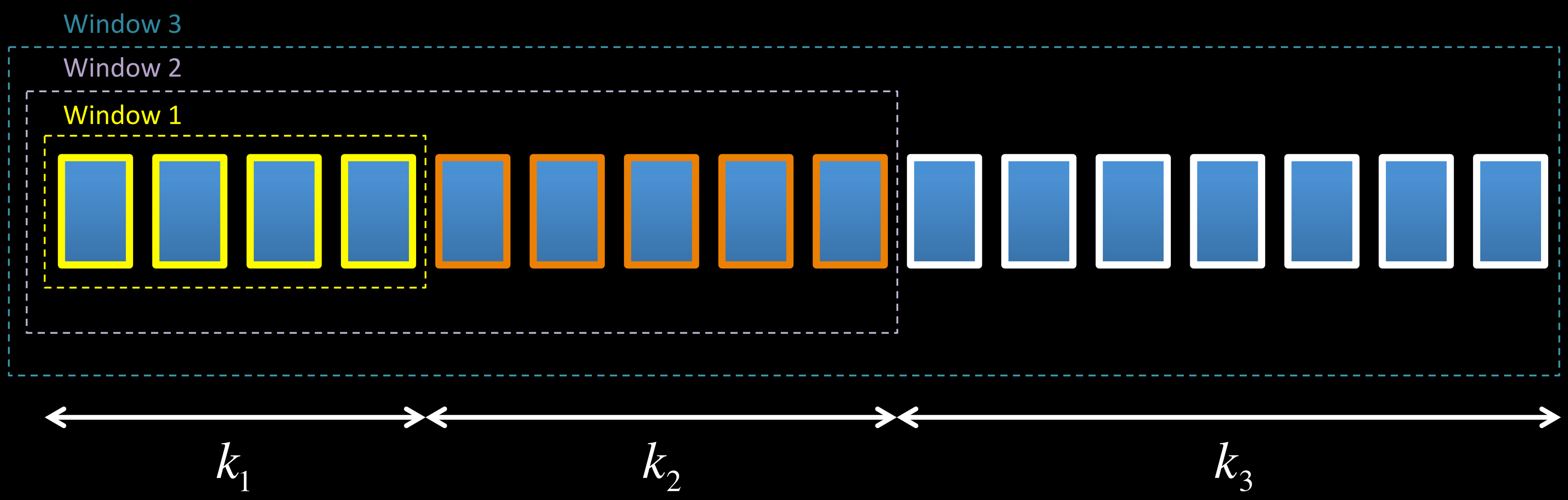

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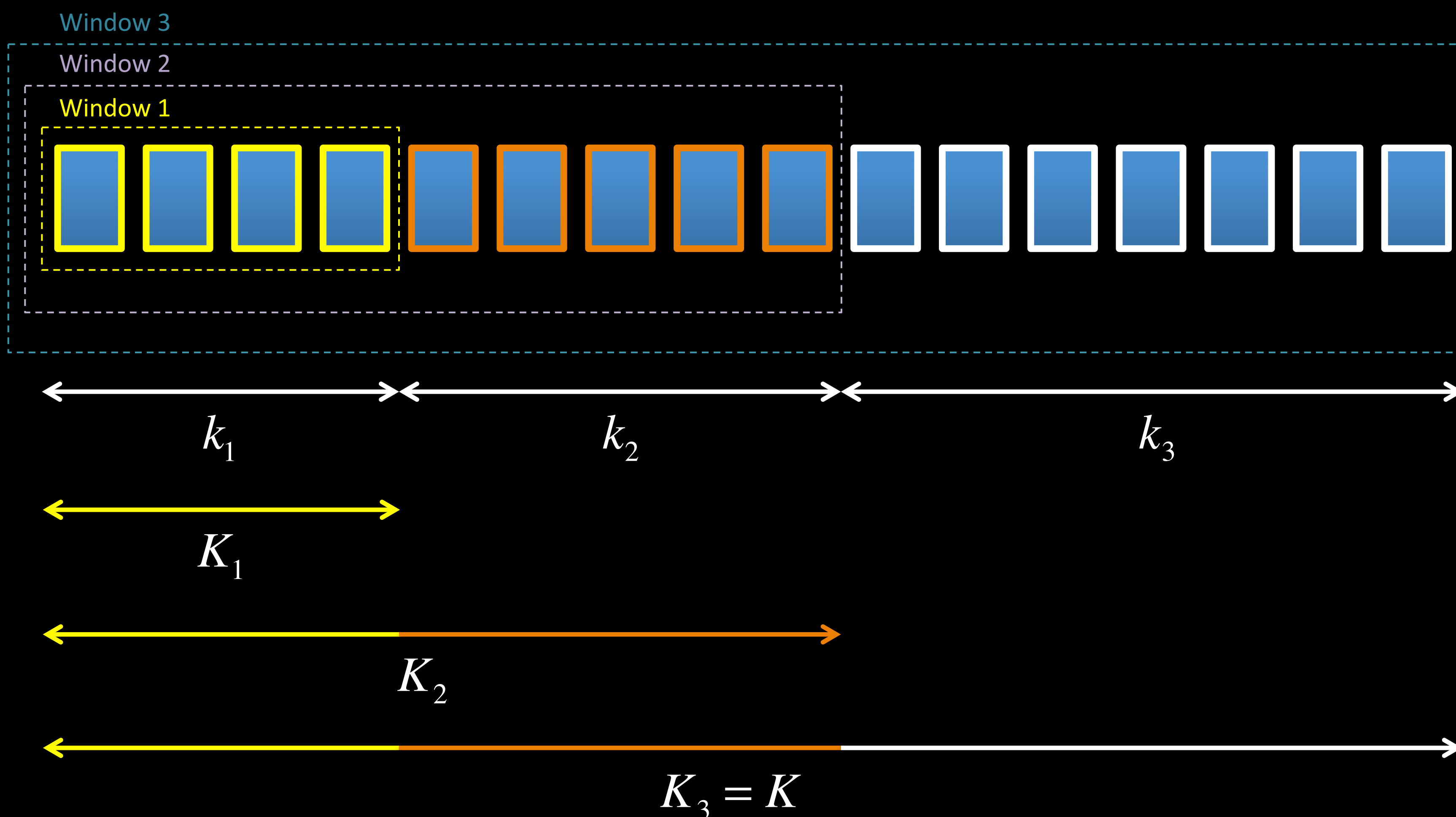

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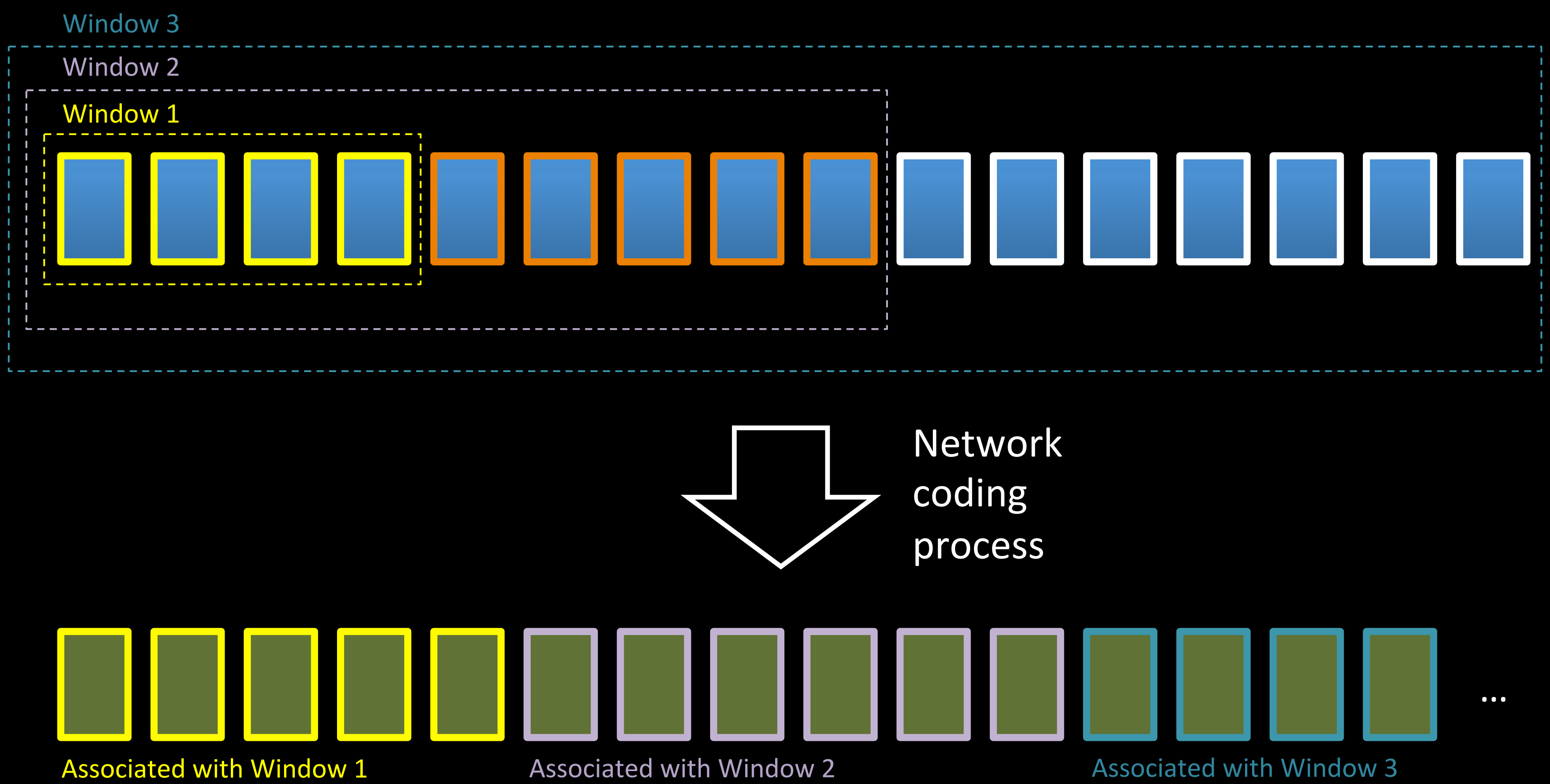

Layered NC: Expanding Window (EW)

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- The probability of the user recovering the first l service layers (or, equivalently, the l-th expanding window), can be expressed as

$$D_{\mathrm{EW}}(N_1, \ldots, N_l) =$$
$$= \sum_{r_1=0}^{N_1} \sum_{r_2=0}^{N_2} \cdots \sum_{r_l=r_{\min,l}}^{N_l} \binom{N_1}{r_1} \cdots \binom{N_l}{r_l} p^{\sum_{i=1}^{l}(N_i - r_i)} (1-p)^{\sum_{i=1}^{l} r_i} g_l(r_1, \ldots, r_l)$$

where $r_{\min,\,l} = K_l - K_{l\text{-}1} + \max(r_{\min,\,l\text{-}1} - r_{l\text{-}1}, 0)$ and $r_{\min,1} = K_1$. The probability that K_l out of the $r_1 + r_2 + \ldots + r_l$ received coded packets are linearly independent is $g_l(r_1, \ldots, r_l)$.

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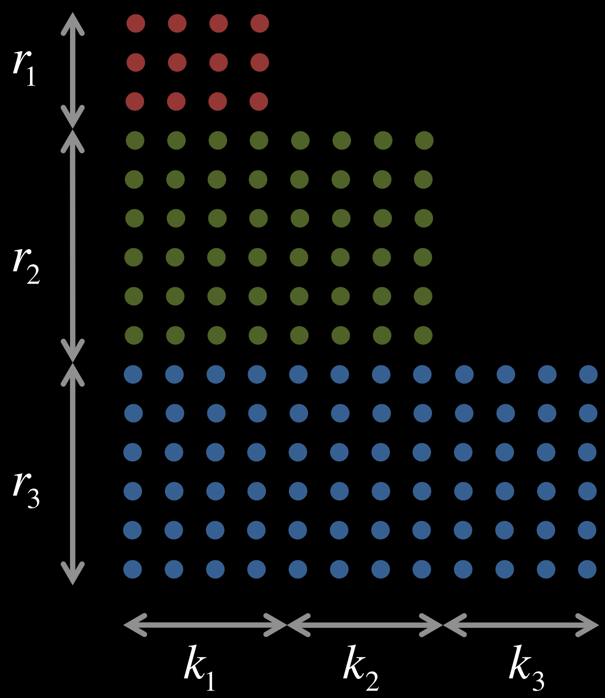
Decoding probability of EW-RLNC

- If a $(r_1+r_2+r_3) \times (k_1+k_2+k_3)$ matrix is the vertical concatenation of three random matrices with dimensions $r_1 \times k_1$, $r_2 \times (k_1+k_2)$ and $r_3 \times (k_1+k_2+k_3)$, what is the probability of the matrix having full rank?



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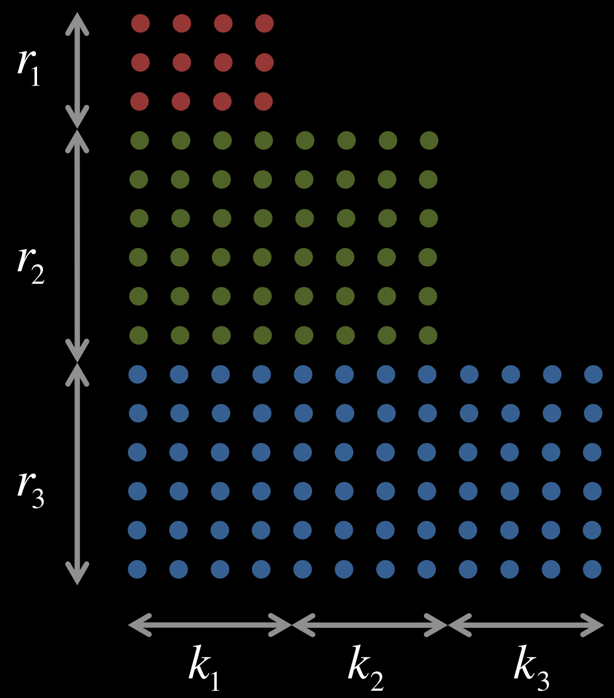

Recall that $w_K(r)$ is the probability that a random matrix of dimensions $r \times K$, for $r \geq K$, has full rank (i.e., the rank is K).

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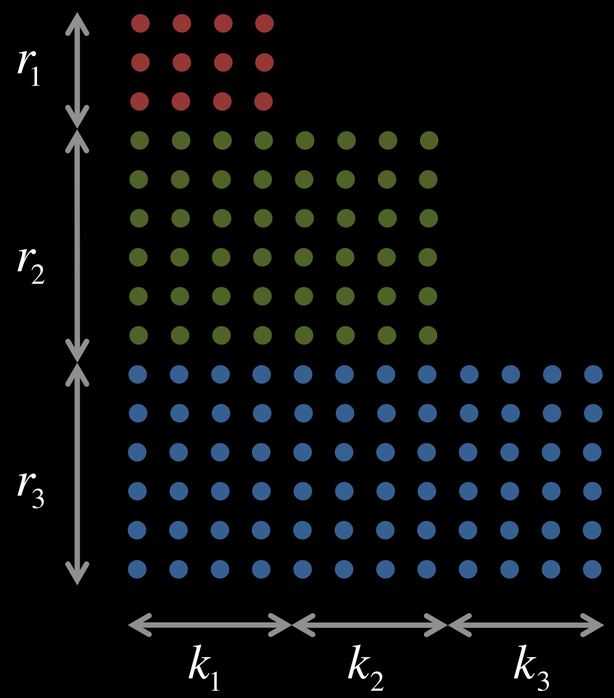

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Let $w_\beta(r, K)$ be the probability that a random matrix of dimensions $r \times K$ has rank β, where $0 \leq \beta \leq \min(r, K)$.

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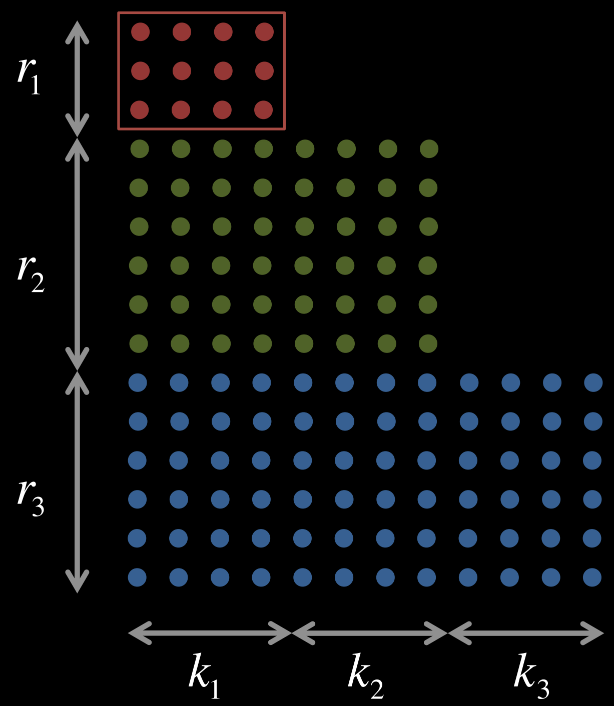

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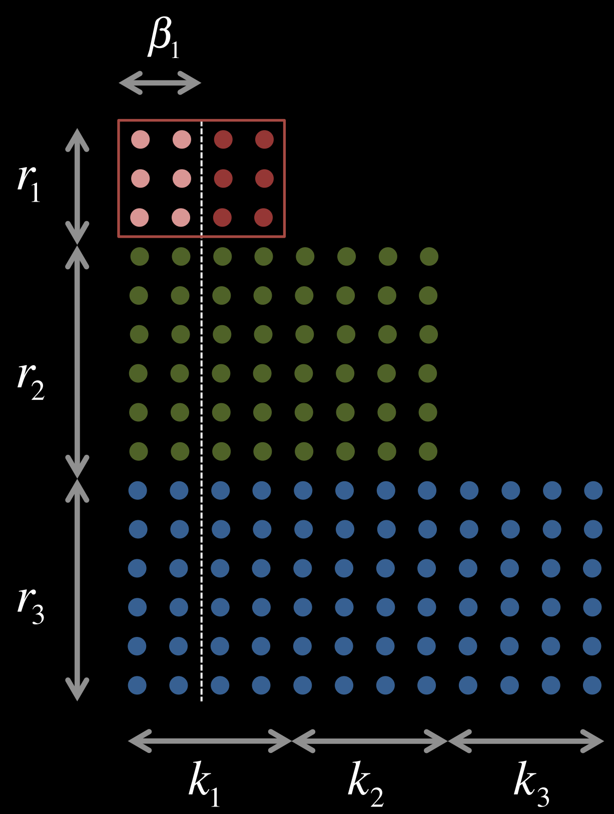

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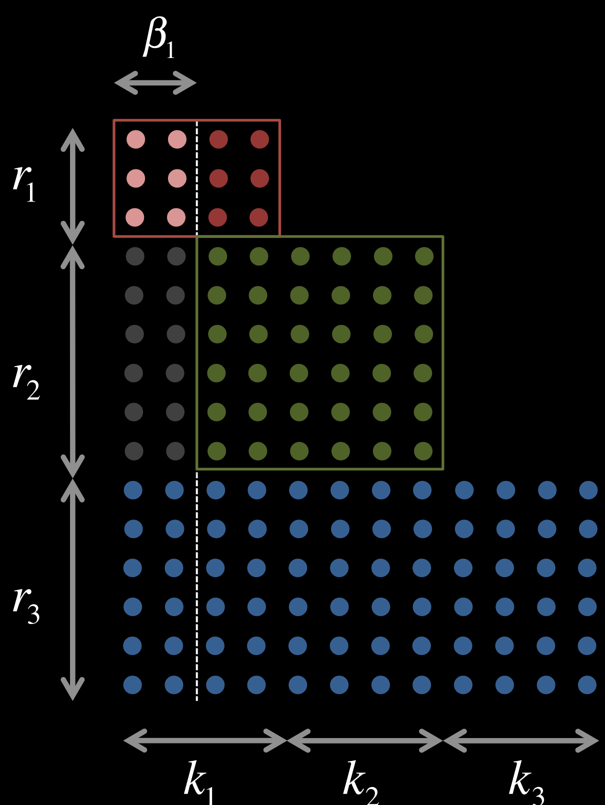

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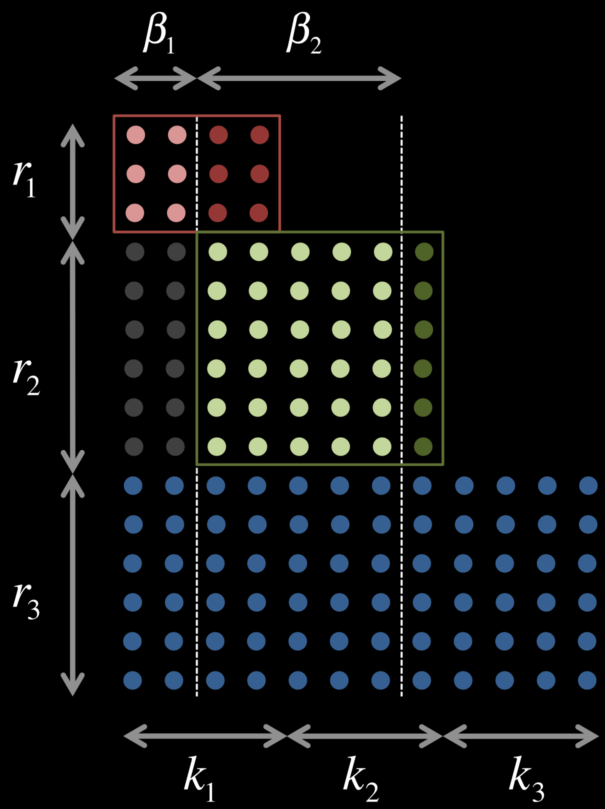

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$$w_{\beta_2}(r_2, k_1 + k_2 - \beta_1)\, q^{r_2\beta_1}$$

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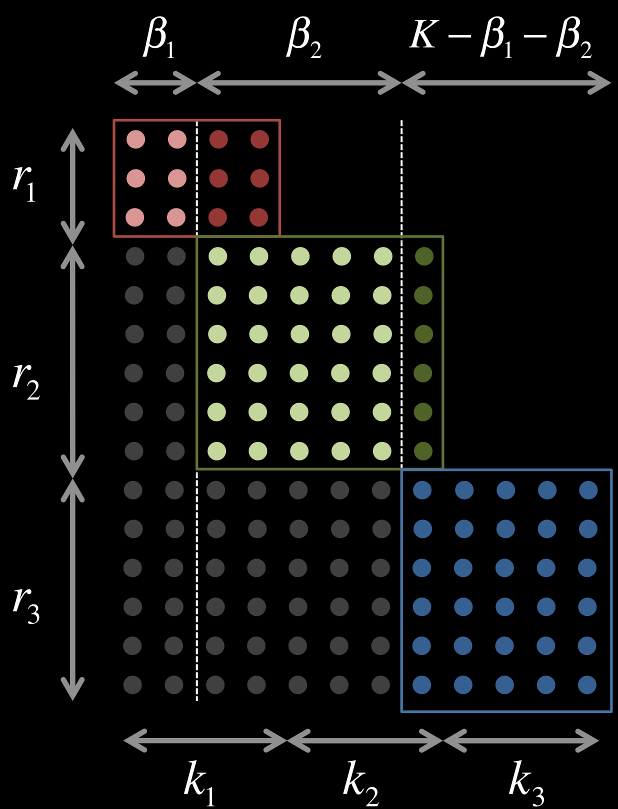

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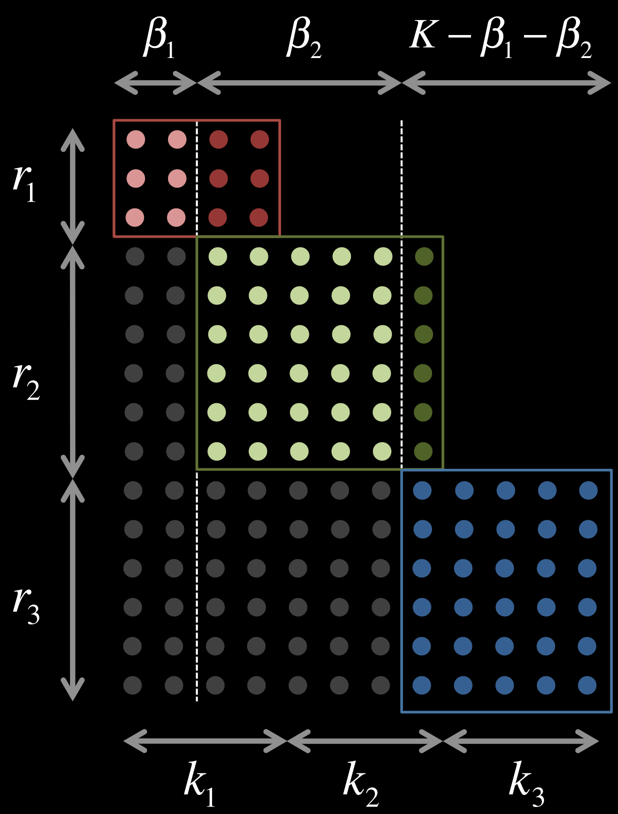

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$$= w_{K-\beta_1-\beta_2}(r_3)\, q^{r_3(\beta_1+\beta_2)}$$

Decoding probability of EW-RLNC

- In general, the probability that K_l out of the $r_1 + r_2 + \ldots + r_l$ received coded packets are linearly independent is

$$g_l(r_1,\ldots,r_l) = \sum_{\beta_1} \cdots \sum_{\beta_{l-1}} \prod_{i=1}^{l} w_{\beta_i}\left(r_i,\ K_i - B_{i-1}\right) q^{\sum_{k=1}^{l-1} r_{k+1} B_k}$$

where: $B_i = \beta_1 + \beta_2 + \ldots + \beta_i$ for $i > 0$ and $B_0 = 0$ for $i = 0$

$$\max\left(0,\ K - B_{i-1} - \sum_{j=i+1}^{l} r_j\right) \le \beta_i \le \min\left(r_i,\ K_i - B_{i-1}\right)$$

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- Note that the probability of an $r \times K$ random matrix having rank β is

$$w_\beta(r,K) = \binom{r}{\beta}_q \frac{w_\beta(K)}{q^{rK}}$$ where $\left(\ \right)_q$ denotes the q-binomial coefficient

Sparse random linear network coding

- So far, we have considered the following cases:
  - Straightforward random linear network coding
  - Systematic RLNC
  - Non-Overlapping Window RLNC (NOW-RLNC)
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- In the aforementioned cases, the elements of the coding matrix were selected uniformly at random from GF(q).
- What if the probability of selecting the zero element is higher (or lower) than the other elements of GF(q)?

Sparse random matrices

$$
\begin{array}{c c}
 & \begin{array}{cccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \ldots & x_{K-1} & x_K \end{array} \\
\begin{array}{c} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ \vdots \\ y_{N-1} \\ y_N \end{array} &
\begin{pmatrix}
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet
\end{pmatrix}
\end{array}
$$

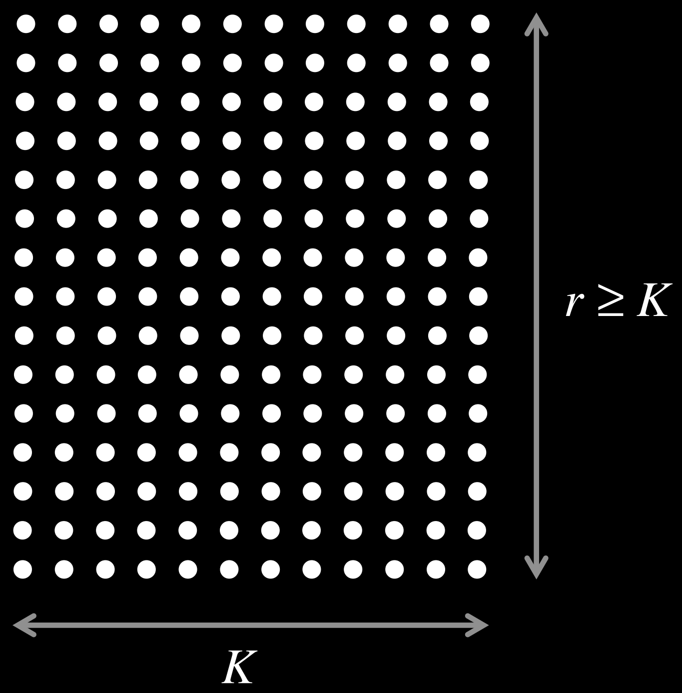
where • is an element of GF(q):

$$
\bullet = \begin{cases} 0, & p \\ 1, & (1-p)/(q-1) \\ \cdots & \cdots \\ q-1, & (1-p)/(q-1) \end{cases}
$$

The probability of a sparse random matrix having full rank has been bounded from below. We are not aware of exact expressions.

Bound on the decoding probability of sparse NC

- The probability of a receiver decoding all of the K source packets, given that $r \geq K$ coded packets have been successfully received, is given by:



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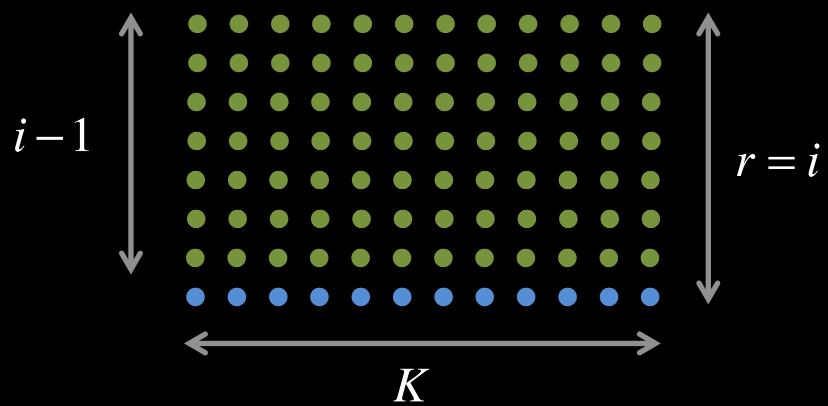

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What is the probability that the i-th row is also linearly independent?

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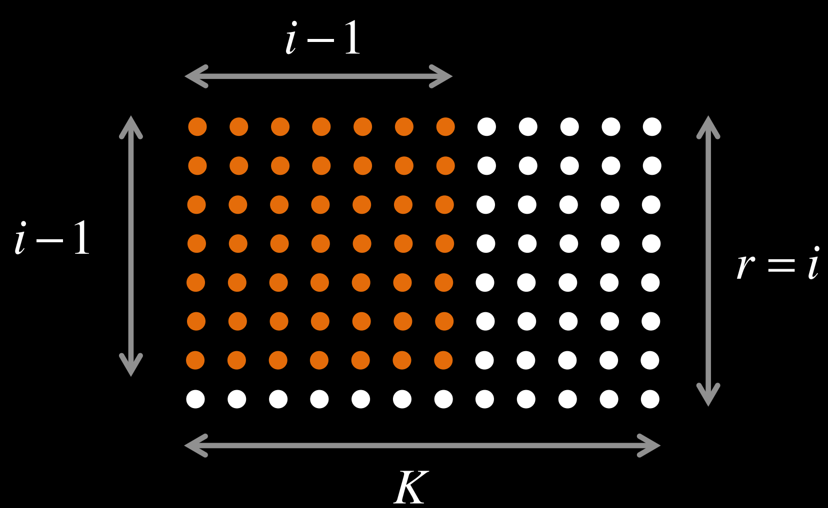

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By elementary row operations, the top $(i - 1) \times K$ sub-matrix can be transformed into a matrix that contains the $(i - 1) \times (i - 1)$ identity matrix.

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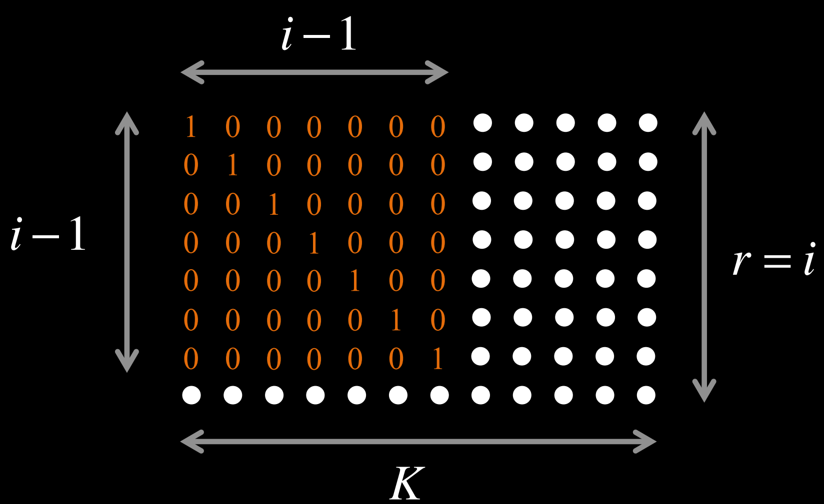

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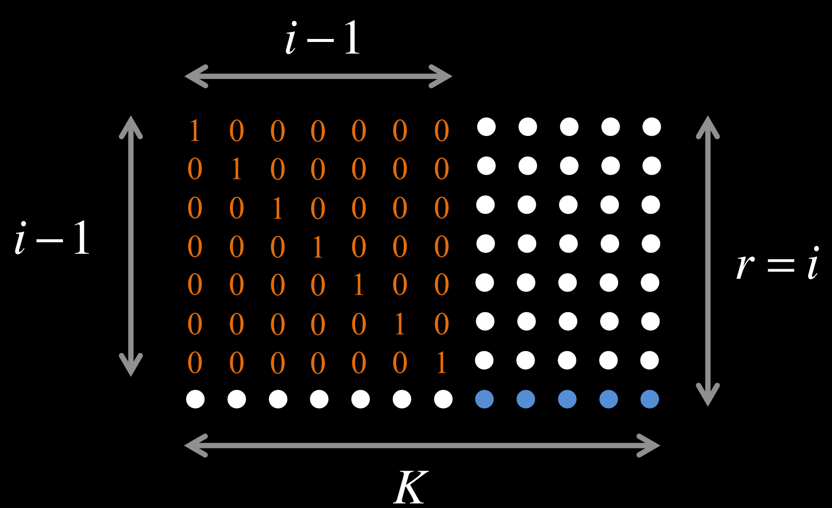

Assume that the top i - 1 rows are linearly independent.

What is the probability that the i-th row is also linearly independent?

The first (i - 1) elements of the i-th row can be arbitrary but the remaining (K - i + 1) elements are uniquely determined by the first (i - 1) elements.

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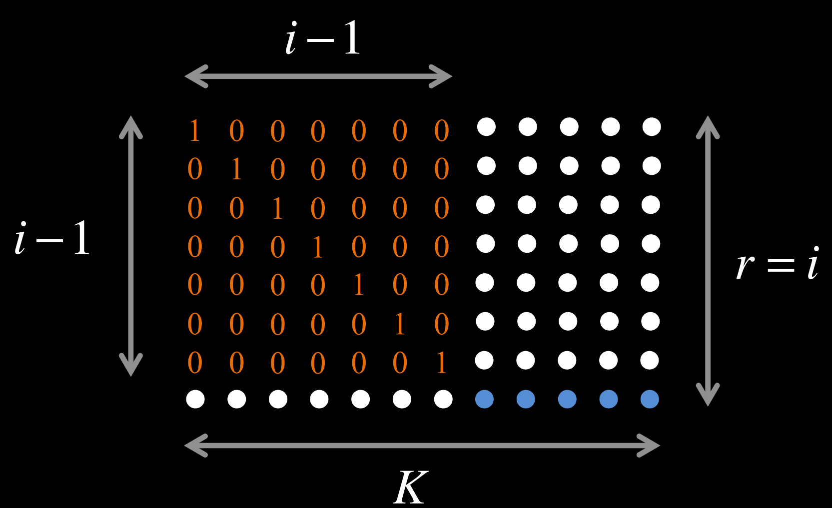

Assume that the top i - 1 rows are linearly independent.

What is the probability that the i-th row is also linearly independent?

The probability that the i-th row is not contained in the subspace generated by the top i rows is at least $1 - \varepsilon^{K-i+1}$, where $\varepsilon = \max((1\text{-}p)/(q\text{-}1), p)$.

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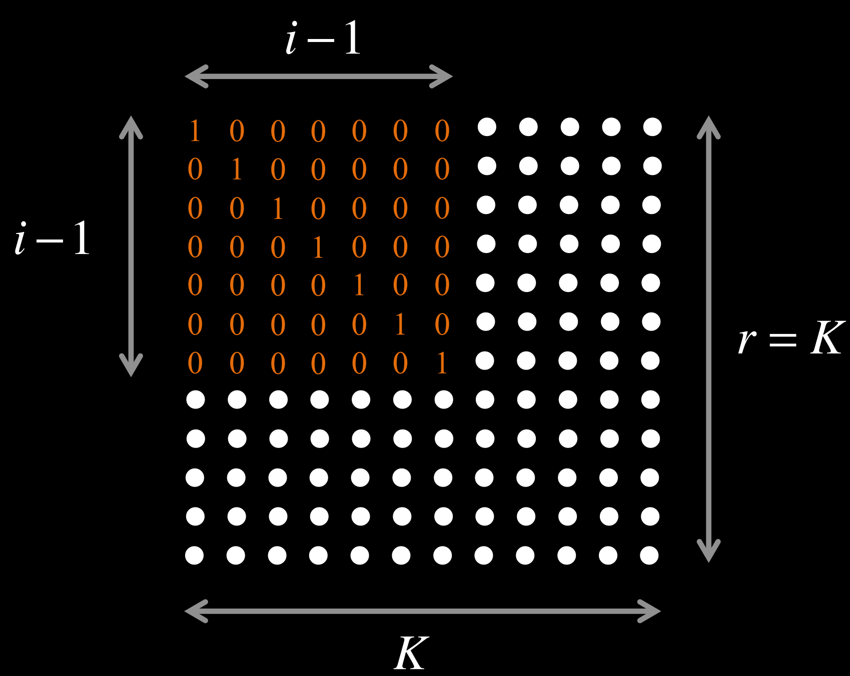

Let $\varepsilon = \max\left(\frac{1-p}{q-1}, p\right)$

Then:

$$\psi_K(r) \geq \prod_{i=1}^{K}\left[1-\varepsilon^{(K-i+1)}\right]$$

$$= \prod_{i=0}^{K-1}\left[1-\varepsilon^{(K-i)}\right]$$

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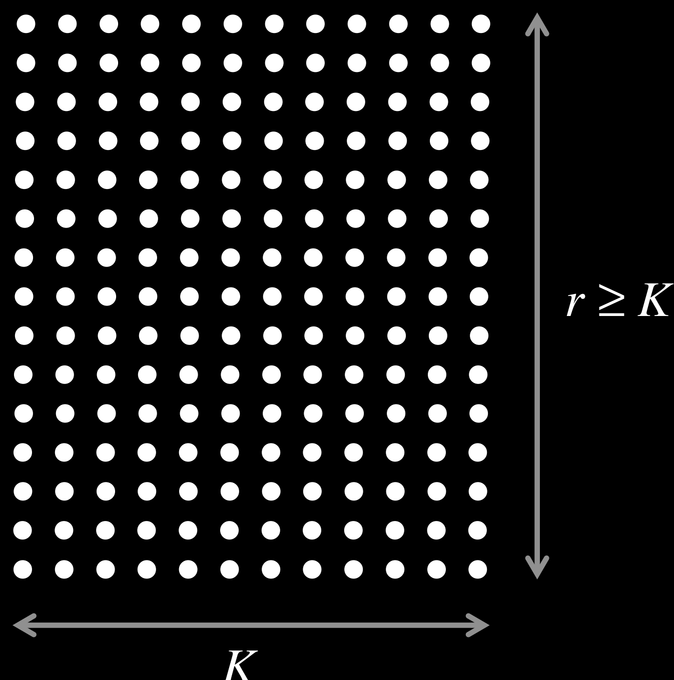

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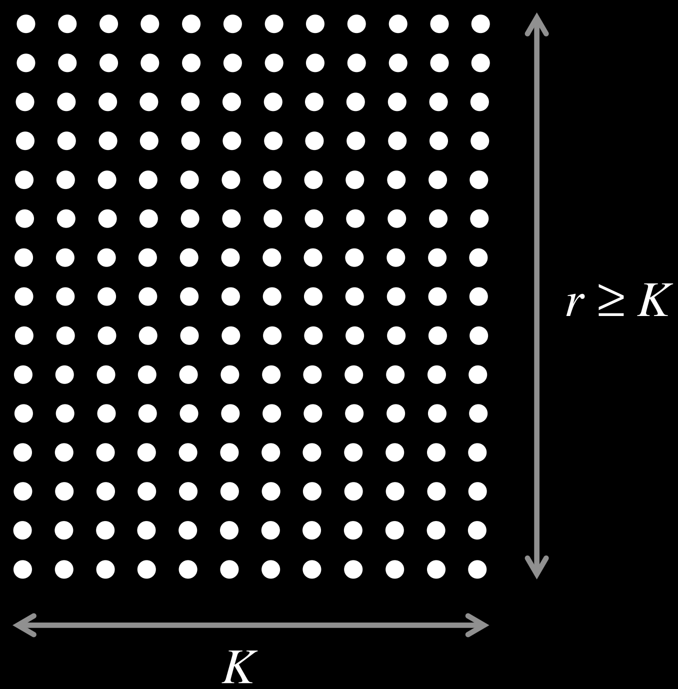

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Then:

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Compare it with

$$w_K(r) = \prod_{i=0}^{K-1}\left[1-(1/q)^{(r-i)}\right]$$

2. Resource allocation for network-coded systems

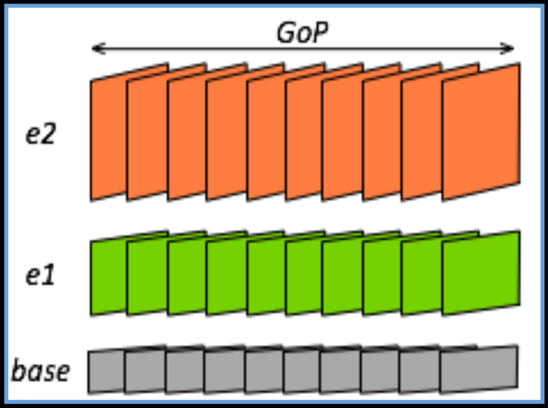
Layered video streams

H.264/SVC video stream formed by multiple video layers:

- the base layer – provides basic reconstruction quality
- $(L-1)$ enhancement layers – gradually improve the quality of the base layer

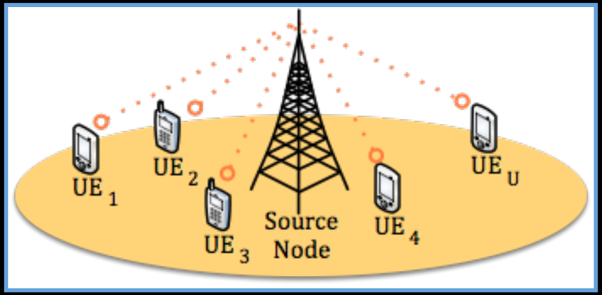
Consider an H.264/SVC video stream:

- It is a GoP stream
- A GoP has fixed number of frames
- It is characterized by a time duration
- It has a layered nature

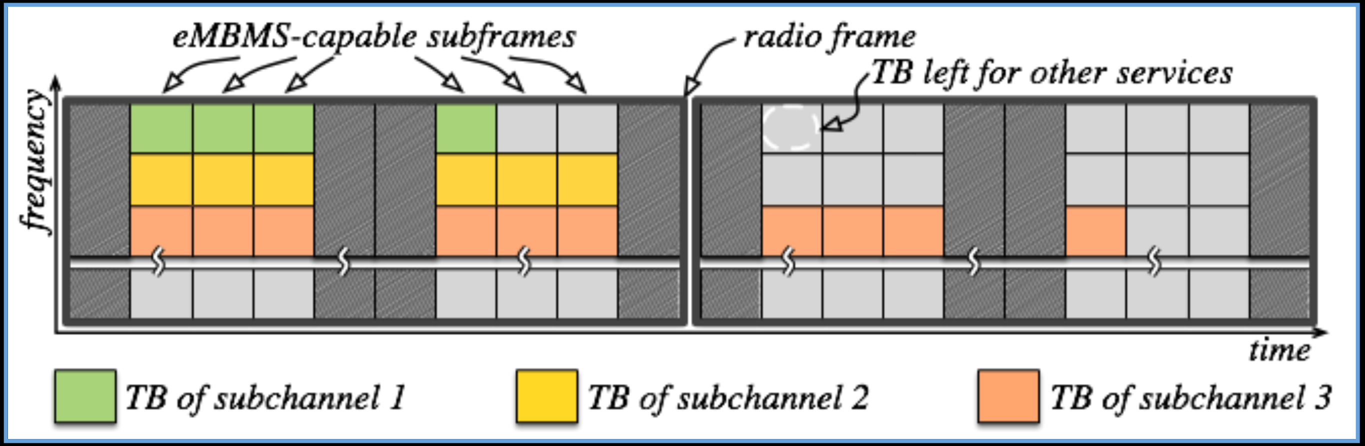


Basic LTE-Advanced system model

- One-hop wireless communication system composed of a single source node (eNB) and U users (UEs).
- Point-to-point communications are managed by the eMBMS framework.

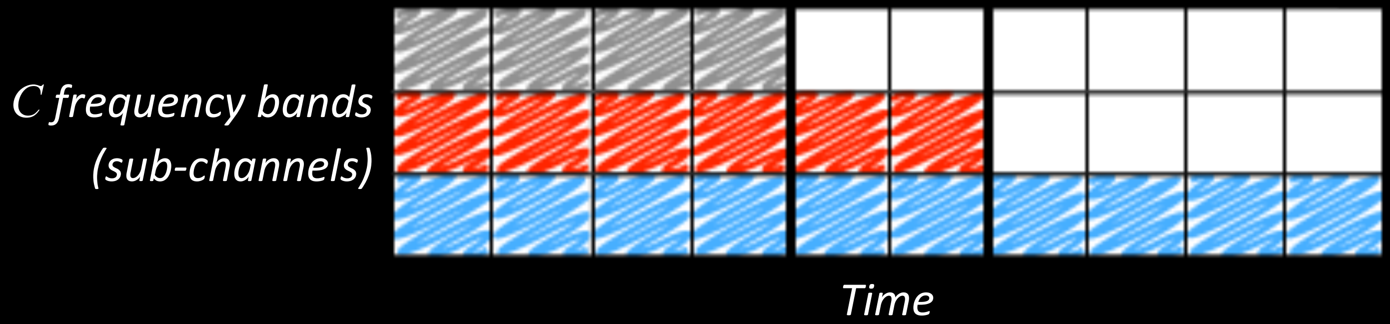


- The downlink of LTE-A adopts an OFDM structure and has a framed nature



Formulation of optimization models

- Abstract structure of transmission medium:



- We consider NOW-RLNC and introduce the following indication variable

$$\lambda_{u,l} = I\left(D_{\mathrm{NOW}}(n_{1,u},\ldots,n_{l,u}) \geq \hat{P}\right)$$

- Similarly, we use the following indication variable for EW-RLNC

$$\mu_{u,l} = I\left(\operatorname*{OR}_{j=l}^{L}\left\{D_{\mathrm{EW}}(N_{1,u},\ldots,N_{j,u}) \geq \hat{P}\right\}\right)$$

Provider-centric optimization models

Proposed mixed allocation (MA) strategies:

(NOW-MA)

$$\min_{\substack{m_1,\ldots,m_L \\ n^{(1,c)},\ldots,n^{(L,c)}}} \sum_{l=1}^{L}\sum_{c=1}^{C} n^{(l,c)}$$

subject to:

$$\sum_{u=1}^{U} \lambda_{u,l} \geq U\ \hat{t}_l \quad \text{for } l = 1,\ldots,L$$

$$m_{c-1} \leq m_c \quad \text{for c} = 2,\ldots,L$$

$$0 \leq \sum_{l=1}^{L} n^{(l,c)} \leq \hat{B}_c \quad \text{for c} = 1,\ldots,L$$

(EW-MA)

$$\min_{\substack{m_1,\ldots,m_L \\ N^{(1,c)},\ldots,N^{(L,c)}}} \sum_{l=1}^{L}\sum_{c=1}^{C} N^{(l,c)}$$

subject to:

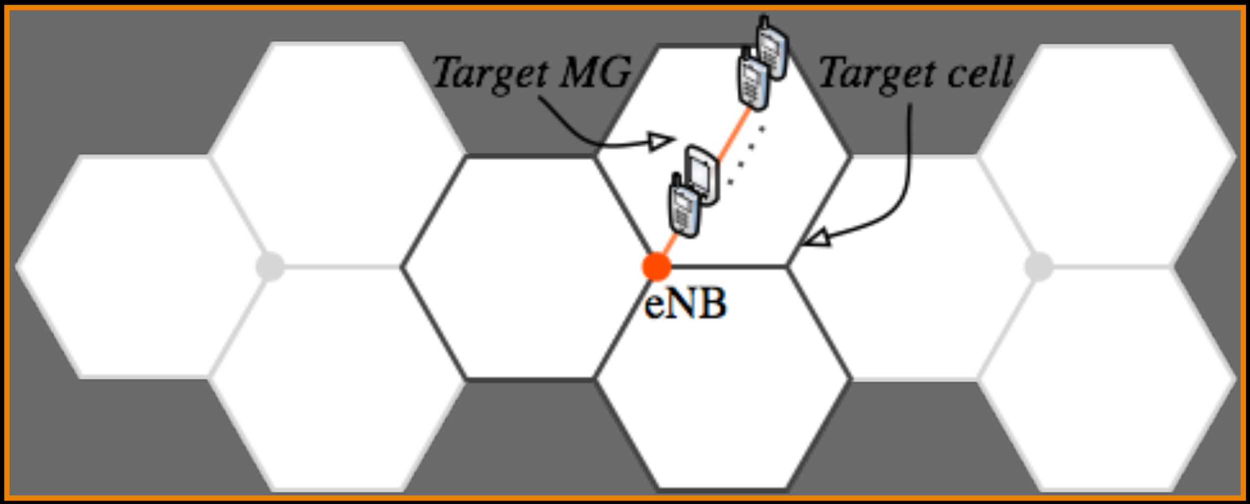
$$\sum_{u=1}^{U} \mu_{u,l} \geq U\ \hat{t}_l \quad \text{for } l = 1,\ldots,L$$

$$m_{c-1} \leq m_c \quad \text{for c} = 2,\ldots,L$$

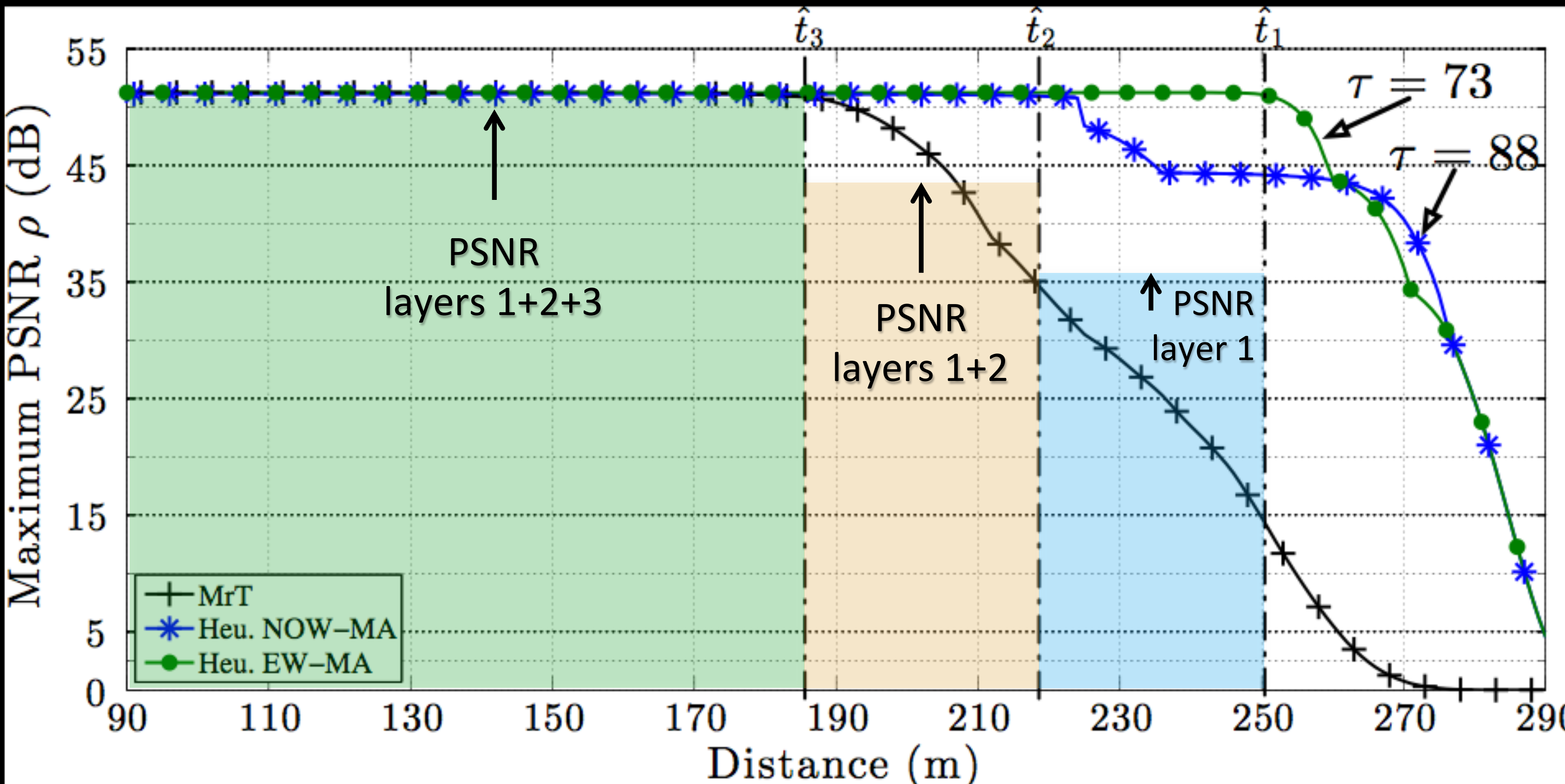
$$0 \leq \sum_{l=1}^{L} N^{(l,c)} \leq \hat{B}_c \quad \text{for c} = 1,\ldots,L$$

Network configuration

- We have considered U=80 users equally spaced and placed along the radial line representing the symmetry axis of one sector of the target cell.
- Node eNB (eNode-B) represents the base station. Users form a Multicast Group (MG).



Analytical results



The performance of the proposed EW-MA is compared to NOW-MA and the state-of-the-art Multi-rate Transmission (MrT). The focus is on GF(2).

User-centric optimization model

- The objective of the previous allocation methods was to minimize the number of transmitted packets, while meeting a minimum set of service level agreements . This is from the point of view of the service provider…
- Best practice for burglars: To steal objects with the maximum value and the minimum weight. Maximizing the profit-cost ratio is the objective of the Unequal Error Protection Resource Allocation Model (UEP-RAM) .

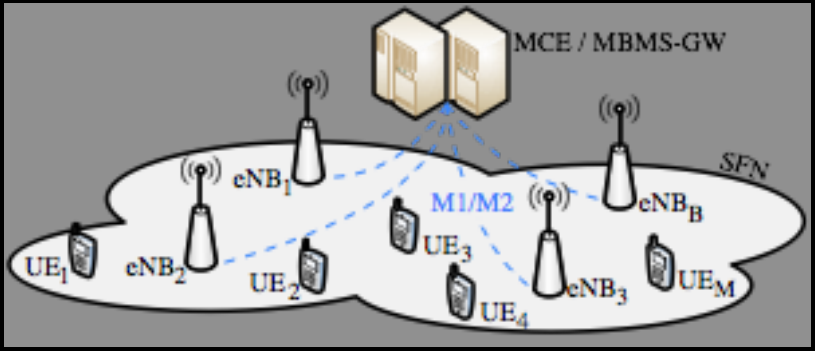
(UEP-RAM)
$$\max_{\substack{m_1,\ldots,m_L \\ N_1,\ldots,N_L}} \sum_{u=1}^{U}\sum_{l=1}^{L}\delta_{u,l} \Big/ \sum_{l=1}^{L} N_l$$

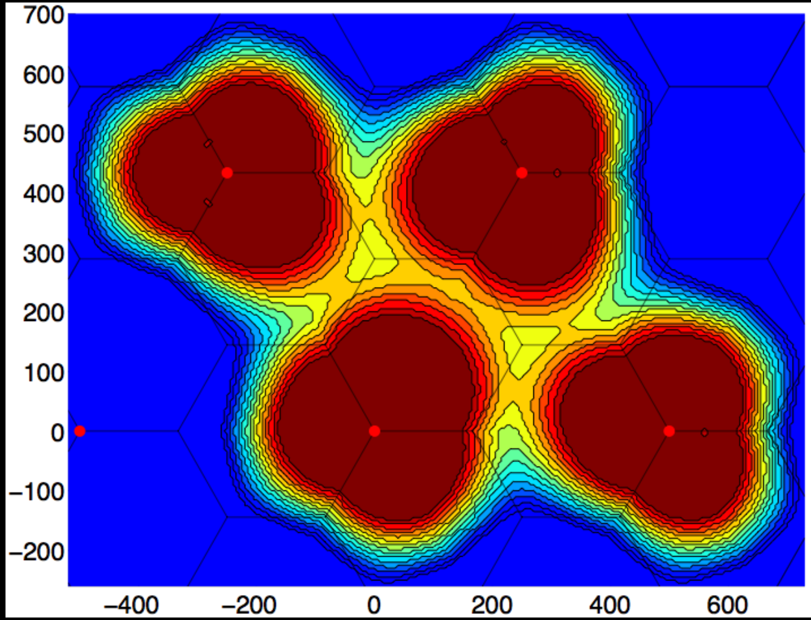
subject to:
$$\sum_{u=1}^{U}\delta_{u,l} \geq U\ \hat{t}_l \qquad l = 1,\ldots,L$$

$$0 \leq N_l \leq \hat{N}_l \qquad l = 1,\ldots,L$$

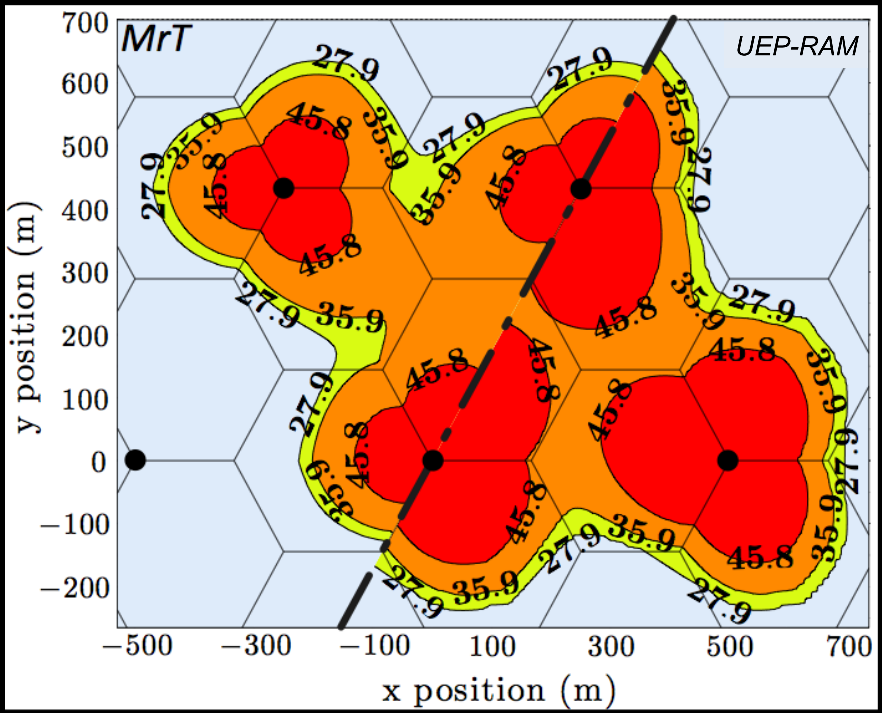
Network configuration

- LTE-Advanced allows multiple contiguous base stations to deliver (in a synchronous fashion) the same services.
- Top image: we considered a Single-Frequency Network (SFN) comprising 4 base stations and 1700 users.
- Bottom image: The distribution of the Signal-to-Interference-plus-Noise Ratio (SINR) in space.

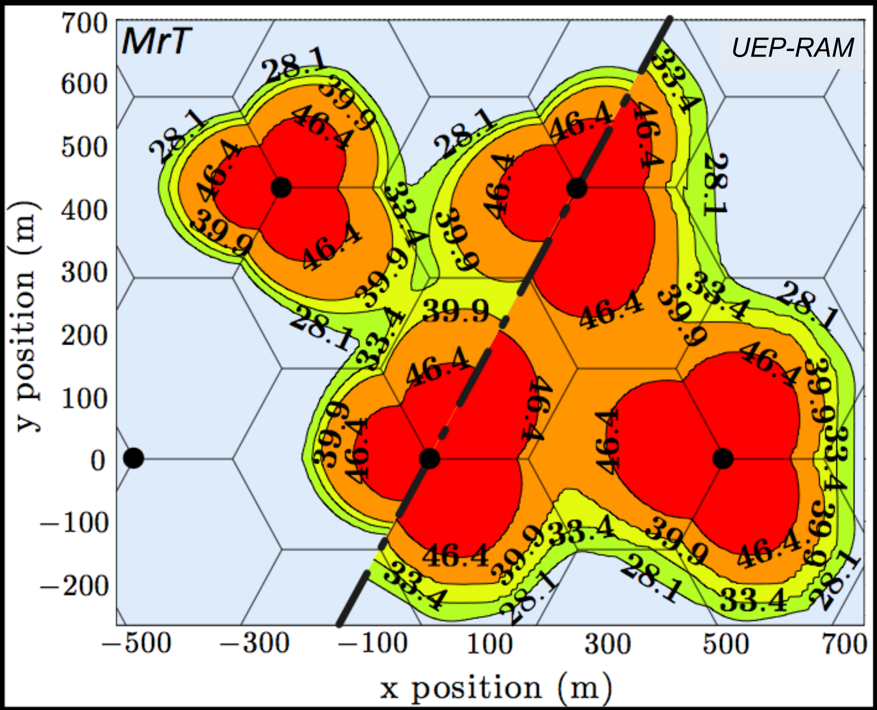




Analytical results



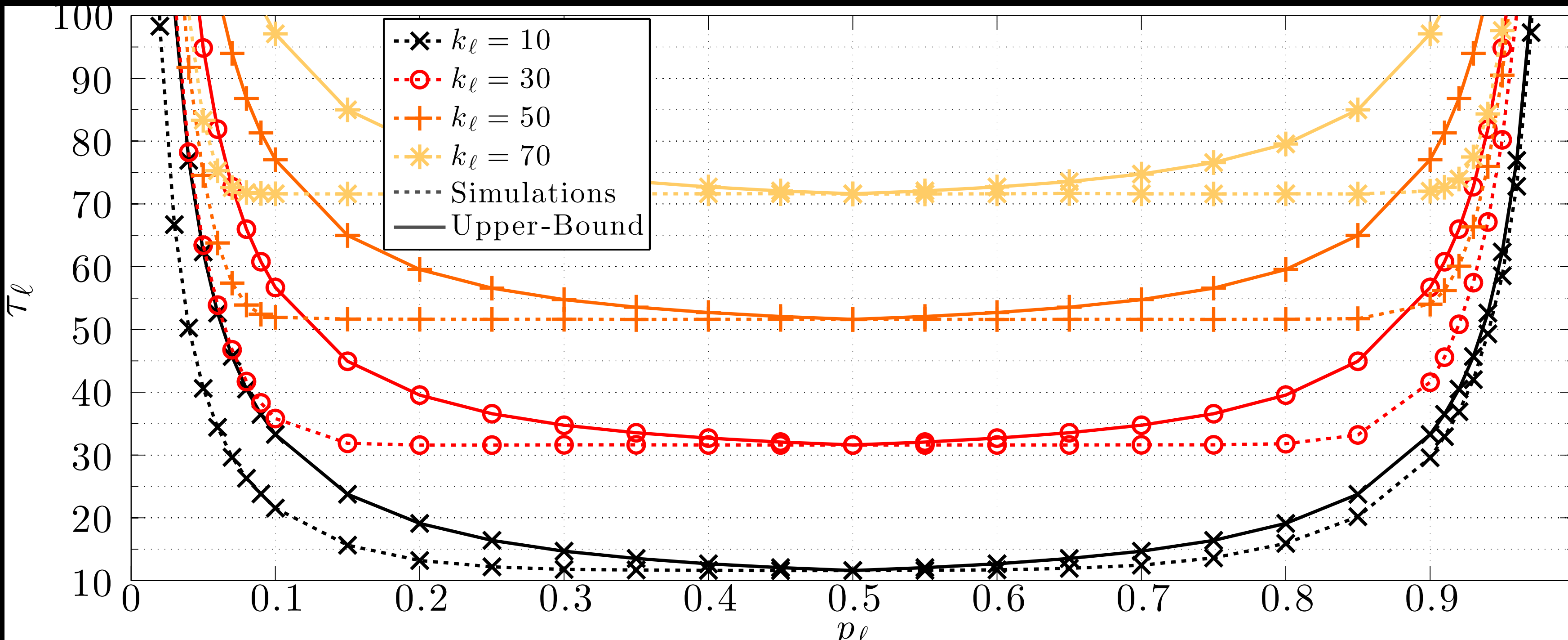
Three-layered service



Four-layered service

The proposed optimization framework based on network coding clearly shows an increase in the coverage of service provider (right-hand side of each figure).

Sparse Random Network Coding



- Results for different numbers of source packets at the transmitter. Upper bounds (solid lines) are compared to simulations (dashed lines).
- Focus on GF(2). An increase in p_ℓ (probability of selecting the zero element) causes an increase in sparsity and a decrease in decoding complexity.
- For 50 source packets and $p_\ell = 0.5$, time to decode=1030 µsec; for $p_\ell = 0.9$, time to decode=352 µsec.

3. Concluding remarks

Concluding remarks

Exact expressions, approximations or even bounds on the decoding probability of standard-agnostic network-coded schemes can be used to optimize the performance of standard-specific systems that employ network coding. More specifically:

- Theoretical analysis is useful for obtaining performance metrics and identifying / quantifying trade-offs between network coding schemes (e.g., systematic vs. straightforward RLNC or EW-RLNC vs. NOW-RLNC).
- Performance metrics can then be used in the definition of resource allocation frameworks, that can jointly optimize system parameters and error control options.
- Efficient heuristic strategies can be developed for the derivation of good quality solutions in a finite number of steps.

Relevant outcomes of the R2D2 project

- A. S. Khan and I. Chatzigeorgiou, "Random block matrix theory for the assessment of network coding techniques", *IEEE Trans. on Commun. (TCOM)*, under review
- A. Tassi, I. Chatzigeorgiou and D. E. Lucani, "Analysis and optimization of sparse random linear network coding for reliable multicast services", *IEEE Trans. on Commun. (TCOM)*, under review
- A. S. Khan, A. Tassi and I. Chatzigeorgiou, "Rethinking the intercept probability of random linear network coding", *IEEE Commun. Letters (COMML)*, vol. 19, no. 10, pp. 1762-1765, Oct. 2015
- A. Tassi, I. Chatzigeorgiou and D. Vukobratović, "Resource allocation framework for network-coded layered multimedia multicast services", *IEEE Journal on Selected Areas in Communications* (JSAC), vol. 33, no. 2, pp. 141-155, Feb. 2015
- A. Khan and I. Chatzigeorgiou, "Performance analysis of random linear network coding in two-source single-relay networks", *Proc. IEEE Int. Conf. on Commun.* (ICC), Workshop on Cooperative and Cognitive Mobile Networks, London, UK, June 2015.
- A. Tassi, I. Chatzigeorgiou, D. Vukobratović and A. L. Jones, "Optimized network-coded scalable video multicasting over eMBMS networks", *Proc. IEEE Int. Conf. on Commun.* (ICC) , London, UK, June 2015.
- A. L. Jones, I. Chatzigeorgiou and A. Tassi, "Binary systematic network coding for progressive packet decoding", *Proc. IEEE Int. Conf. on Comm.* (ICC) , London, UK, June 2015.

Project website

http://www.lancs.ac.uk/~chatzige/R2D2/index.html

R2D2

HOME ABOUT TEAM RESEARCH NEWS CONTACT

THE R2D2 PROJECT

R2D2 was an 18-month research project supported by EPSRC under the First Grant scheme (EP/L006251/1), which was completed in July 2015. This website will keep being updated until all project-related outcomes have been disseminated. Alas, this project did not aspire to develop a prototype of R2D2, the famous droid in the Star Wars films. The focus of the project was on network error control and the research problem under investigation was not in the least less challenging or exciting!

Long-Term Evolution (LTE), the dominant system for fourth generation (4G) networks, has introduced state-of-the-art **fountain coding** to support wireless content streaming and downloading. Even though fountain coding can be combined with **collaborative network coding** to reduce network overhead and improve bandwidth efficiency, there is great scope for tailoring transceiver designs to the requirements of content distribution and for developing radically new paradigms capable of supporting high quality media in next generation systems. **The aims of the R2D2 project can be summarised into the following two points:**

The saga continues…